Analysis of the 2009 REV Race Car Suspension

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Project Summary

The purpose of this project is to analyse and optimise the suspension system of the 2009 REV race car. An effective suspension system acts as a vibration filter, and protects the vehicle from excitation caused by irregularities in the road.

Suitable coil springs are to be selected for the front and rear of the car, to ensure that both ends do not oscillate violently out of phase after going over a bump. The Half Car Model, which has four degrees of freedom, can be used to simulate the vehicle driving in a straight line. By providing excitation to the wheels, the model can be used to show the vertical accelerations of the sprung mass and the front and rear unsprung masses at different frequencies. The selected spring combination should give the least possible vertical accelerations of the sprung and unsprung masses. A common method of determining the extent of driver discomfort is to measure the vertical acceleration of the sprung mass.

It is also important that the suspension is able to damp the unsprung masses adequately to prevent wheel hop. Wheel hop is the vertical vibrating motion of the wheel between the sprung mass and the road, and has the capacity to impede handling and performance.

The Half Car Model will also be used to determine the effects of changing the amplitude of the assumed road, and changing the damping rate of the wheel.

Letter of transmittal

Peter Corke
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23rd October, 2009-10-20

Professor David Smith
Dean
Faculty of Engineering, Computing and Mathematics
University of Western Australia
35 Stirling Highway
Crawley, WA, 6009

Dear Professor Smith,

I am pleased to submit this thesis, entitled **“Analysis of the 2009 REV Race Car Suspension”**, as part of the requirement for the degree of Bachelor of Engineering.

Yours sincerely

Peter Corke
20139918

Acknowledgements

I would like to thank the following people who have helped make this project possible.

All of the REV team, for being great people to work with.

Lachlan Tomlin, for the hours that he put into the race car.

Chris Rowles and Grant Keady, for giving me good advice with my computer modelling.

My friends and family, for the support that they have given me this year.

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1. Introduction

Project objectives

The original plan for the REV team was to receive the 2001 Motorsport SAE car in ‘rolling chassis’ form at the beginning of the year, and to transform it into a battery-powered race car with hub motors. The aim of this thesis was going to be to redesign the suspension system of the vehicle, making it more suitable to the new weight distribution caused by replacing the petrol motor with a battery cage, batteries and hub motors. Unfortunately, due to significant time delays, the ‘rolling chassis’ was not received until the mid year holidays. Therefore, in order to get a drivable car constructed by the end of the year, as few alterations as possible were made to the car, and, consequently, this thesis became concerned with analysing and optimising the existing suspension system of the car instead of designing a new one.

About midway through second semester, it was decided that trying to design and install a hub motor setup complete with pulleys into each of the existing rear uprights of the car was too difficult. Therefore, it was decided that inboard motors would be used as this approach allowed for much more space and easier installation.

Definitions

a

‘a’ is the distance between the front track and the centre of gravity (CG)

b

‘b’ is the distance between the rear track and the centre of gravity (CG)

CG

CG is an acronym for 'centre of gravity'.

Sprung mass

The sprung mass is comprised of parts of the vehicle that are supported by the suspension, such as the chassis, driver, battery cage, batteries and motors. The sprung mass moves when the springs are deflected (Waldron & Kinzel 2004).

Unsprung mass

The unsprung mass is comprised of parts of the vehicle that are not supported by the suspension, such as the wheels, tyres and uprights (Waldron & Kinzel 2004).

Wheelbase, l

The wheelbase (l) is the distance between the front track and the rear track

<u>Axes</u>

Below is a diagram showing the pitch, roll and yaw axes of a vehicle. The Half Car Model later in this paper is concerned with the pitch axis.

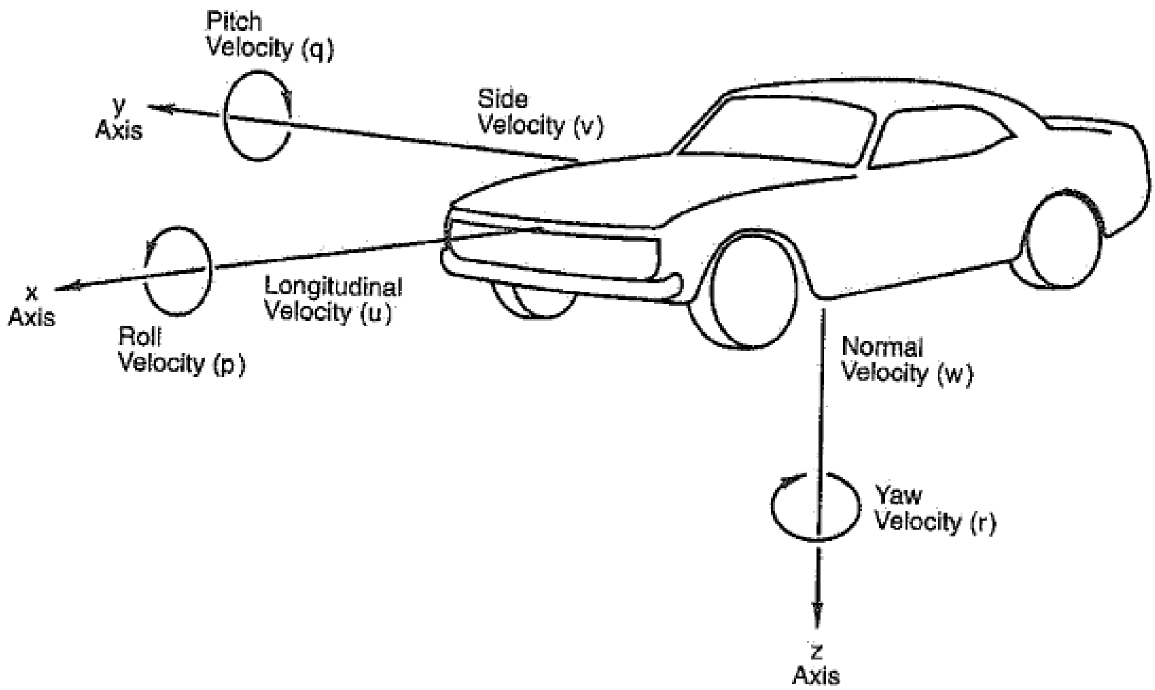


Figure 1.1: Pitch, roll and yaw axes (Milliken & Milliken 1995)

Camber

Below is a diagram showing the camber angle.

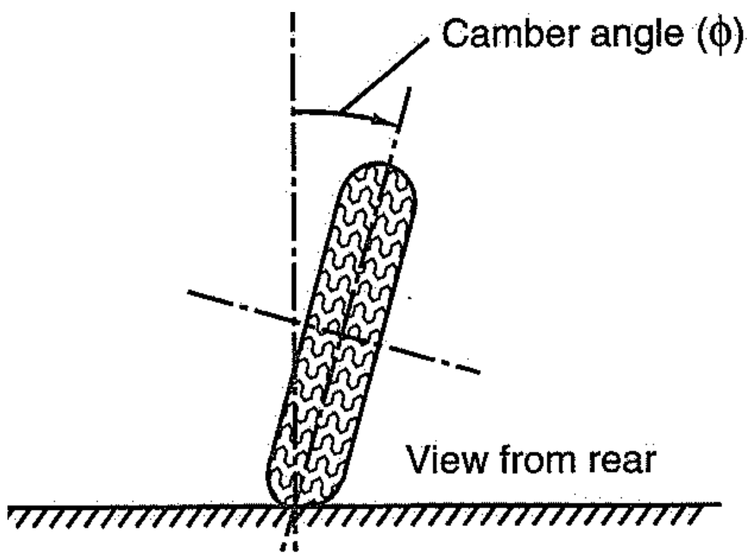


Figure 1.2: Camber angle (Milliken & Milliken 1995)

Below is a diagram showing a negative camber angle of both wheels.

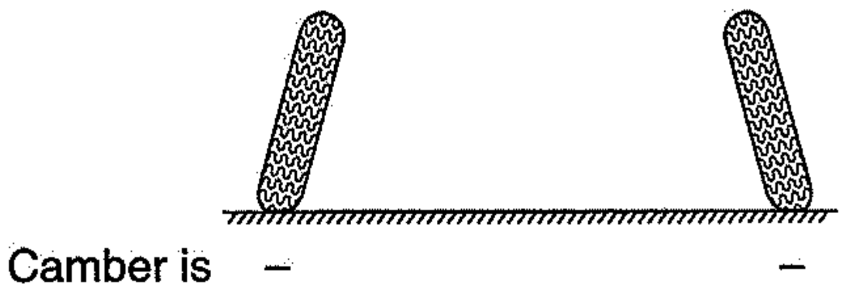


Figure 1.3: Negative camber (Milliken & Milliken 1995)

Toe

Below is a diagram showing both toe in and toe out. Please note the oval shape represents the driver at the front of the car.

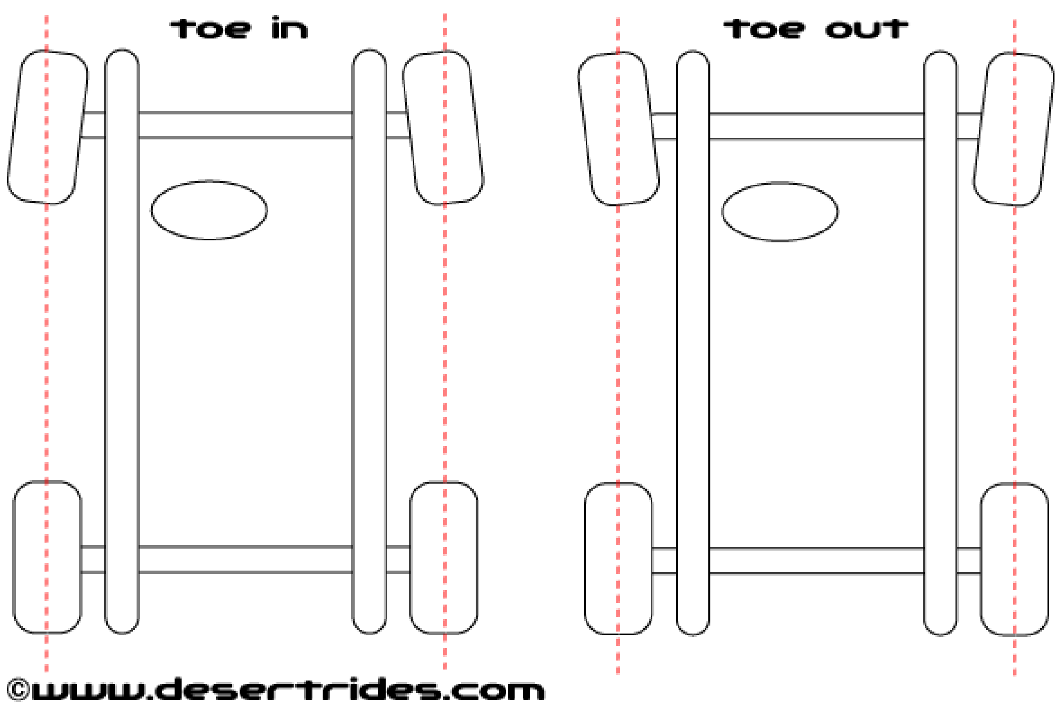


Figure 1.4: Toe in and toe out (Desert Rides 2007)

Importance of suspension

The key requirement of a suspension system is to ensure that all tyres, especially the two attached to the driving wheels, maintain the greatest possible contact with the road. The suspension should protect the vehicle from jolts in the road, and provide stability and roadholding throughout the entire power range of the motors. While cornering, the wheels, especially the outside wheels, should remain as upright as possible. An effective suspension system is very much a compromise between roadholding and ride. The stability of a racing car is influenced by many factors, including:

- Vehicle weight
- Vehicle weight distribution
- Sprung to unsprung weight ratio
- Location of the centre of gravity
- Roll centre heights on the front and rear suspensions
- Wheelbase and track
- Suspension layout
- Steering geometry
- Loading condition
- Characteristics of the tyres, including pressure and tread depth
- Wheel and tyre balance

A successful suspension system generally has non-linear camber change, which means that camber is not directly dependent on vehicle roll (Costin & Phipps 1962).

<u>Wishbone independent suspension</u>

Wishbone independent suspension systems do not encounter as many problems as beam axles.

1. The front end does not experience shimmy, which means that the steering requires no damping. This allows the driver to have a better "feel" for the handling of the car, which is important for race car driving. Also, bump steer can be entirely eradicated at any particular steering angle.

2. Unsprung mass is heavily reduced. Since there is much less mass moving up and down with the wheels, it is easier to maintain contact between the tyres and the road, especially on uneven surfaces.
3. The rear wheels can be cambered, which can increase the cornering power of the tyres. Also, any amount of toe-in or toe-out can be provided.

   (Aird 1997)

The advantage with wishbone suspension systems is that they are normally built with a steering linkage that is symmetrical about the centreline of the vehicle. This means that any "inaccuracies" in the linkage will not cause the vehicle to veer off unnecessarily (Milliken & Milliken 2002).

Equal length and parallel wishbones

The main disadvantage of having equal length and parallel wishbones is that the wheels become cambered the wrong way when the chassis rolls in cornering. To ensure that the treads of the tyres remain flat on the road while cornering, a substantial amount of negative camber can be incorporated into the wheels. This, however, means that when the vehicle is traveling forward, the tyres will be running on their edges. This causes the tyre edges to wear and overheat, and does not allow for much traction when accelerating or braking (Aird 1997).

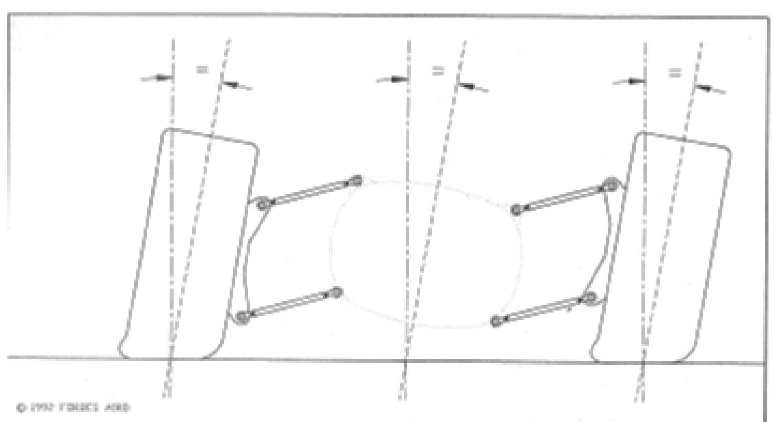


Figure 1.5: Equal length and parallel wishbones in roll (Aird 1997)

Using equal length and parallel wishbones also means that as the suspension moves vertically in bump and rebound, the wheels will rise and fall without cambering. This completely eliminates the problem of gyroscopic "kick", which occurs when a rotating wheel is tilted, although it introduces a new problem. The vertical motion of the wheel is accompanied by some sideways motion, referred to as scrub, and this generates "kickback" in the steering. This is due to the line of the axis the wheel steers around crossing the ground ahead of the tyre contact patch's centre (Aird 1997).

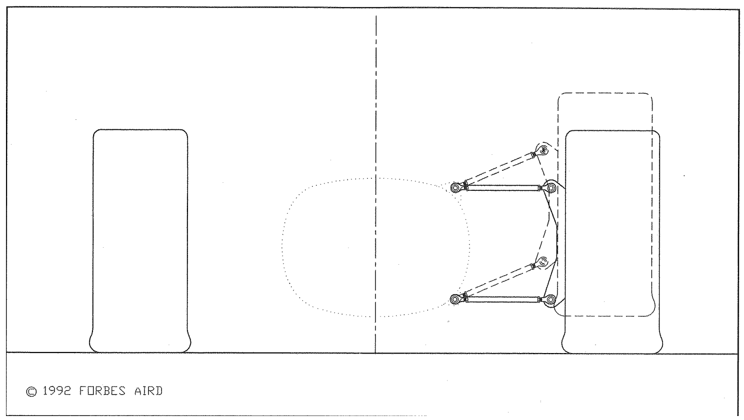


Figure 1.6: Equal length and parallel wishbones in bump (Aird 1997)

Unequal length wishbones

The REV car has independent suspension with unequal length wishbones. By using unequal length wishbones, that is, by making the top wishbone shorter than the bottom one, it is possible to combine small amounts of camber and scrub. Having small amounts of camber and scrub prevents the detrimental effects of having large quantities of either by itself (Aird 1997).

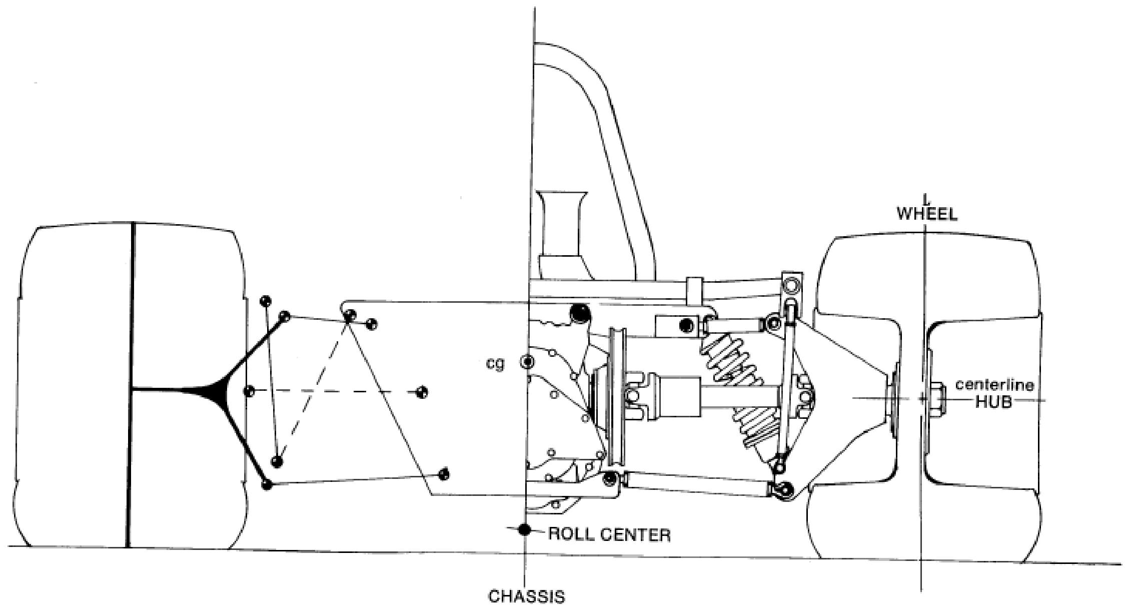


Figure 1.7: Independent suspension with unequal length wishbones (Smith 1978)

Unsprung mass oscillations

The oscillations of the unsprung mass can cause potential problems. These oscillations include:

1. Shimmy
2. Wheelfight
3. Wheel hop

Wheels that are mounted to independent suspension linkages have an advantage over wheels that are attached to beam axles, because, in theory, they have only one degree of freedom, as opposed to six. However, in racing cars, it is essential to avoid too rigid a limitation of the wheel motion, which therefore establishes additional degrees of freedom. Vibrations in the various modes can sometimes supply energy to each other, if they are coupled in a particular way. Of the listed varieties of oscillation, only shimmy is indefinitely self-sustaining on a smooth road (Milliken & Milliken 2002).

Shimmy

Shimmy is considered the most hazardous type of unsprung mass oscillation, and is prevalent in cars where the front wheels are connected via a beam axle. The shimmy cycle begins by one of the front wheels hitting a chance bump in the road, or by an imbalance in one of the front wheels. This causes one front wheel to be flattened into the road, while the other wheel is in the air. The positions of the wheels are then reversed. This cycle is infinitely self-sustaining on a smooth road, and generally occurs at higher speeds.

Shimmy can cause many problems, such as the hands of the driver being shaken off the steering wheels, a tyre blowing or coming off its rim, and damage of steering and suspension linkages. If the driver applies the front brakes during the shimmy cycle, the frequency of the shimmy decreases while its amplitude increases. This can cause the car to become out of control.

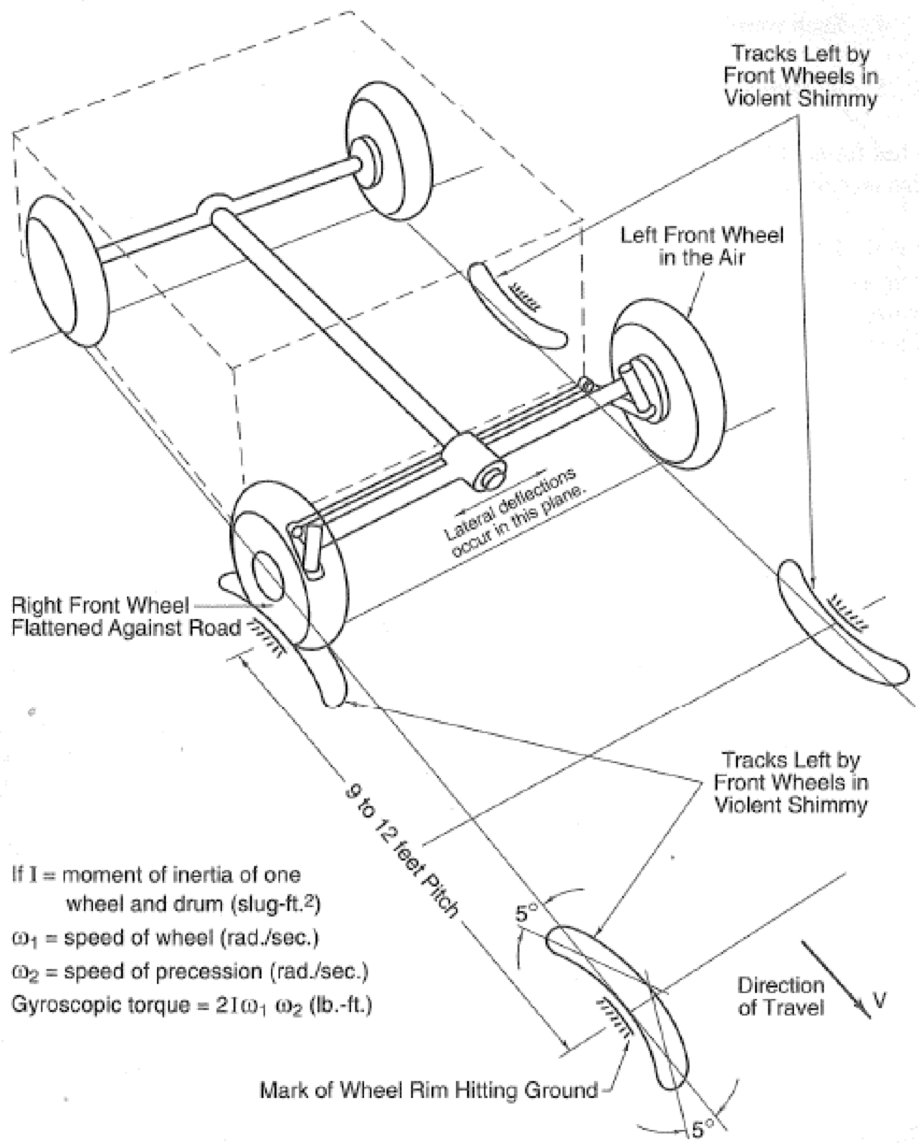


Figure 1.8: A severe shimmy cycle (Milliken & Milliken 2002)

The most effective approach in preventing shimmy is to use independent front suspension, as well as having a torsionally stiff car frame. By placing the sprung mass of the car between the front wheel mountings, the gyroscopic torque of the shimmy cycle is unable to generate the high frequency required for shimmy (Milliken & Milliken 2002).

It is highly unlikely that a shimmy cycle will develop on the REV car, as it has independent suspension at both ends of the car. The chassis, which is a part of the sprung mass, is torsionally stiff, and is manufactured from mild alloy steel 4140. It is fair to state that the chassis is “over-engineered”.

Wheelfight

SAE Vehicle Dynamics Terminology defines wheelfight as “a rotary disturbance of the steering wheel produced by forces acting on the steerable wheels” (Milliken & Milliken 2002). Wheelfight is unlike shimmy as it is not self-sustaining. Instead, it relies on constant excitation supplied by irregularities in the road.

Any complex analysis of wheelfight requires a detailed investigation of the steering gear’s elastic resonance. Pictured below is a steering wheel test rig.

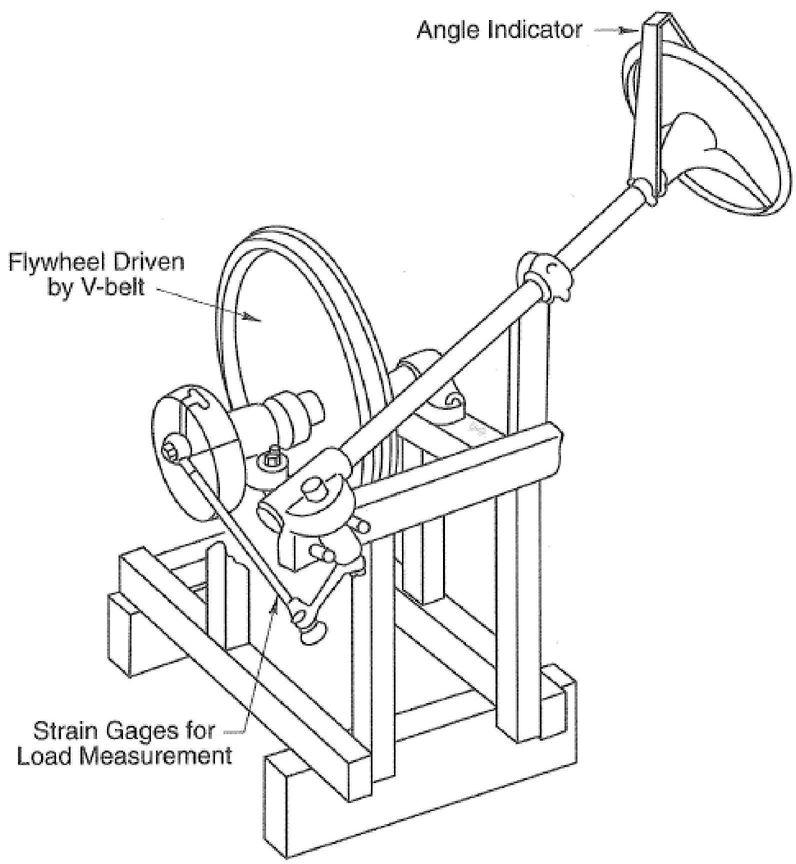


Figure 1.9: Steering gear test rig (Milliken & Milliken 2002)

Wheelfight generally takes place when the frequency of the steering gear is close to the frequency at which wheel hop occurs.

This means that preventative measures against wheelfight can be implemented in either of two locations: at the steering gear or at the steerable wheels (front wheels).

To assist in preventing wheelfight at the steering gear:

i. increase the steering wheel's moment of inertia
ii. reduce the steering wheel shaft diameter

To assist in preventing wheelfight at the steerable wheels,

i. attach rubber mountings to the lower control arms
ii. have suspension linkages designed so that as the front wheels rise they toe out
iii. make the wheel centres move backwards as the wheel rises by sloping the wheel arm pivots
iv. induce a negative swing arm effect (please refer to Appendix A)

It is unlikely that wheelfight will pose a threat to the performance of the REV car, as the steering gear will probably not reach high enough frequencies (wheel hop frequency). To verify this, the natural frequency of the steering gear would have to be determined, however, this is beyond the scope of this project.

Wheel hop

Vehicle ride has two modes: they are primary ride (often simply referred to as ride) and secondary ride. Primary ride is the lower frequency oscillation of the sprung mass, while secondary ride is the higher frequency oscillations of the unsprung masses. Secondary ride is sometimes referred to as wheel hop, which is defined as “the vertical oscillatory motion of a wheel between the road surface and the sprung mass” (Milliken & Milliken 2002).

Too much wheel hop has adverse effects on the handling and stability of the car. Therefore it is important to appropriately damp the unsprung masses, and also the sprung mass, so that the driver is comfortable.

Wheel transmissibility is defined as the ratio of the vertical wheel motion to road input amplitude. Generally transmissibility under 2.5 will not permit enough wheel hop to hinder handling (Milliken & Milliken 1995).

2. Model Formulation

Car information

The total mass of the car when it loaded with a 75kg driver is 274 kg. There are two inboard motors that are mounted to the chassis above the rear track that have a combined mass of 25kg, while the battery cage and batteries are located just behind the driver's seat and have a combined mass of 55kg. The car has an approximate mass distribution of 48% at the front and 52% at the rear.

Below is a screen shot from a computer model of the car that was created using a software package called Solid Works.

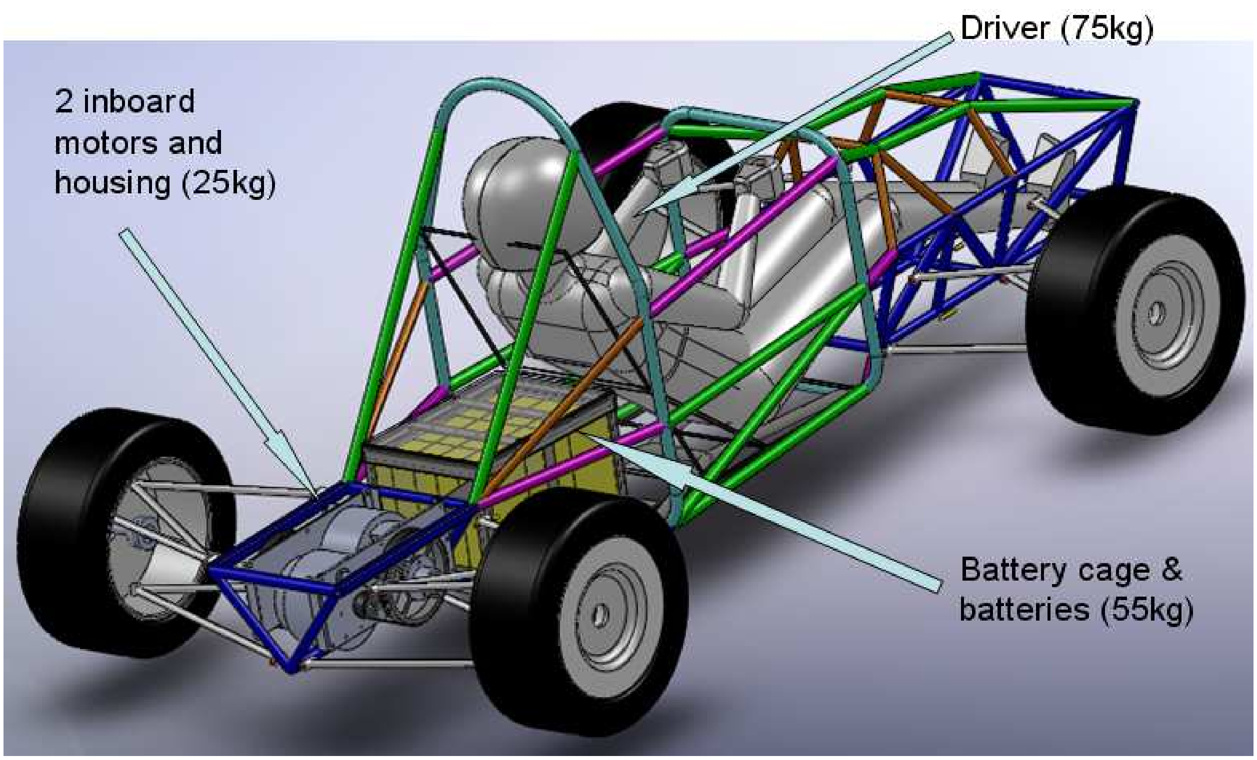


Figure 2.1: Solid Works model of the car (Ivanescu & Tomlin 2009)

The Solid Works software was used to calculate the mass moment of inertia of the sprung mass about its pitch axis through its centre of gravity. This value was found to be 616.363 kg m^2.

The important information about the car is expressed in the table below.

Information	Symbol	
Wheelbase	l	1.85m
Distance from CG to front track	a	0.959m
Distance from CG to rear track	b	0.891m
Sprung mass	M	202kg
Front sprung mass	M_f	97.29kg
Rear sprung mass	M_r	104.71kg
Unsprung mass (same at front and rear)	m	18kg
Mass moment of inertia (about pitch axis through CG)	I	616.363kg m^2

Table 2.1: Important information about the car

Please refer to Appendix B to see the method used for calculating a and b.

Coil springs

Coil springs are the most commonly used type of springs in racing cars with independent suspensions. They can be used in both compression and extension (tension). The helix is the most widespread shape of coil spring, in which the mean diameter of the coil remains constant (Milliken & Milliken 1995).

Below is a diagram of a helix-shaped coil spring.

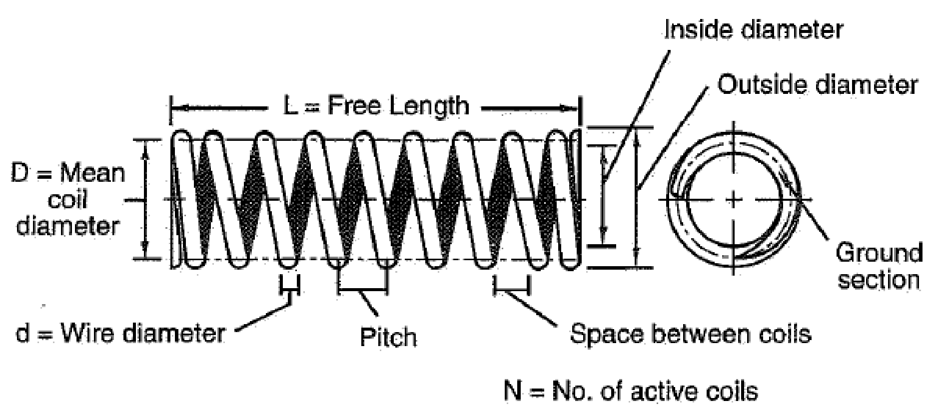


Figure 2.2: Helix-shaped coil spring (Milliken & Milliken 1995)

Fox Vanilla RC Shox

The shock absorbers used in the REV car suspension are of the Fox Vanilla RC Shox variety, which are commonly used on mountain bikes.

The Fox Vanilla RC is a telescopic type monotube damper, which means that its damping force is due to oil being forced through an orifice at high pressures. It has an external tank of pressurised nitrogen gas, which is increasingly compressed as the piston rod moves further up the cylinder (Guzzomi 2004). Below is a diagram of the Fox Vanilla RC damper.

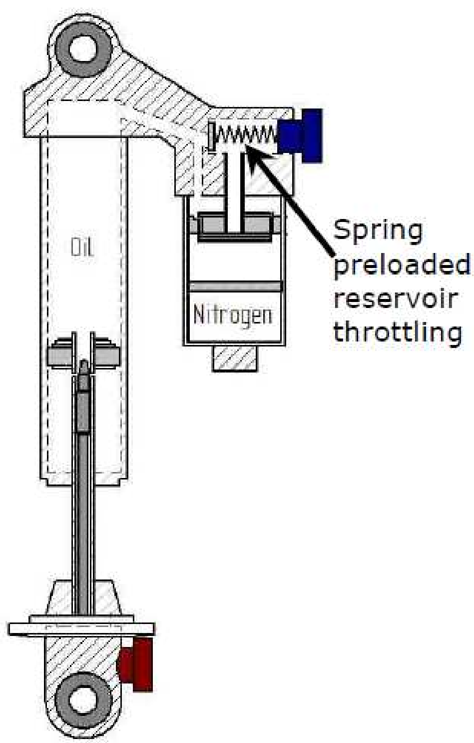


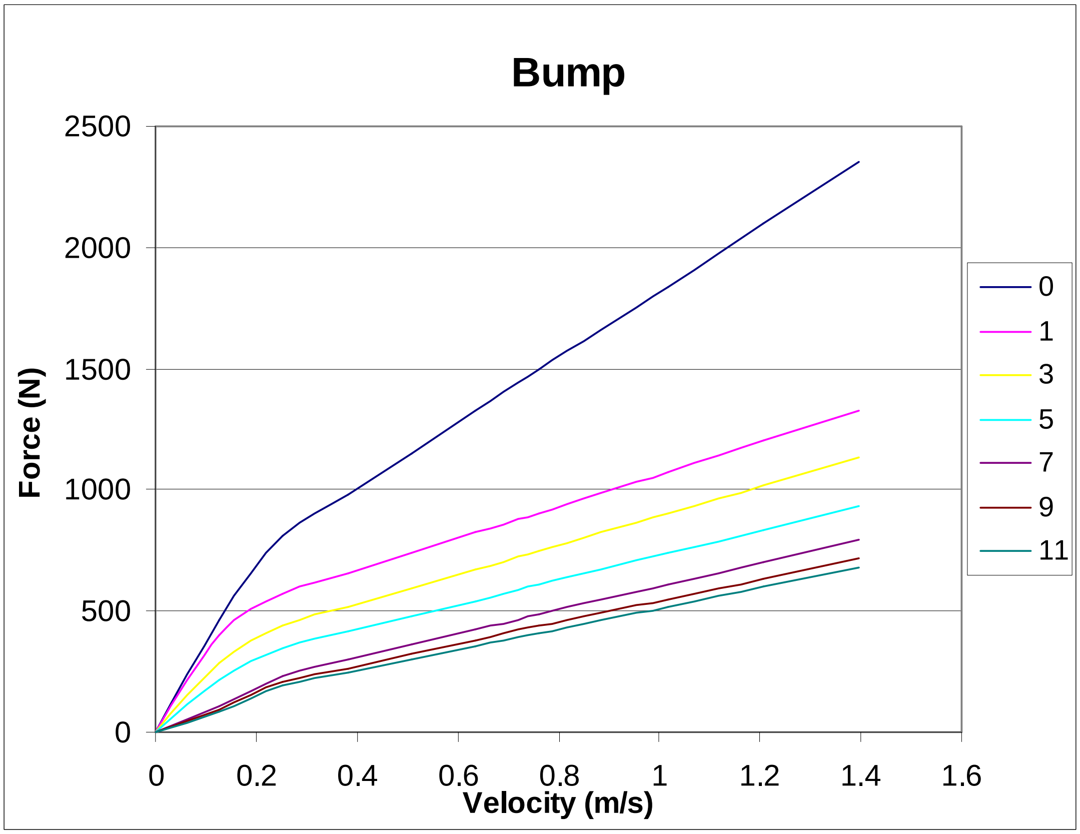
Figure 2.3: Fox Vanilla RC damper (Finlayson 2003)

The Fox Vanilla RC damper has a casing that is manufactured out of hard, anodised aluminium, which increases wear resistance. It has a 25mm bore and has a mass of just over 315 g.

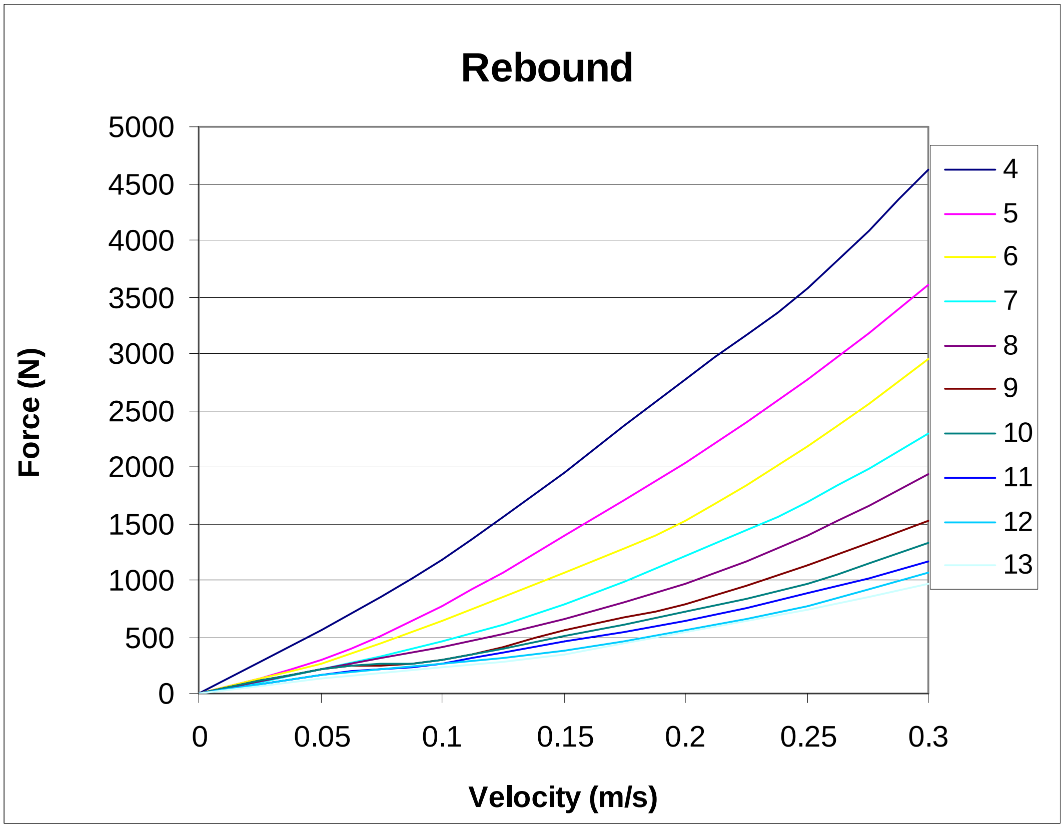
The damper is adjustable in both bump and rebound. To modify the bump settings, the spring preload on the relief valve can be altered (see Figure 2.3). A high spring preload will result in higher damping rates in bump. To modify the rebound settings, there is an adjustable needle valve that controls piston bypass in both bump and rebound, although it has only a small effect on bump damping (Guzzomi 2004).

Adjustments in bump and rebound are done in increments of ¼ turns, or "clicks".

The graphs below show force (N) vs. velocity (m/s) for the Fox Vanilla RC damper in both bump and rebound. The numbers in the legend refer to the number of "clicks" open.



Graph 2.1: Fox Vanilla RC damper in bump



Graph 2.2: Fox Vanilla RC damper in rebound

It is noticeable in the "bump" graph that there is a distinct kink in each of the curves, which occurs at around the same velocity, regardless of the number of "clicks". This is due to the presence of a "blow-off" valve, which causes the bypass of a proportion of the oil at high pressures. The reason for this is to avoid very high damping forces at high velocities (Milliken & Milliken 1995). From the graph, it can be seen that "blow-off" occurs at approximately 0.2 m/s.

<u>Quarter Car Model</u>

The Quarter Car Model is used to simulate one corner of the car. It has two degrees of freedom: the vertical movement of the unsprung mass, and the vertical movement of the sprung mass.

Below is a diagram of the Quarter Car Model.

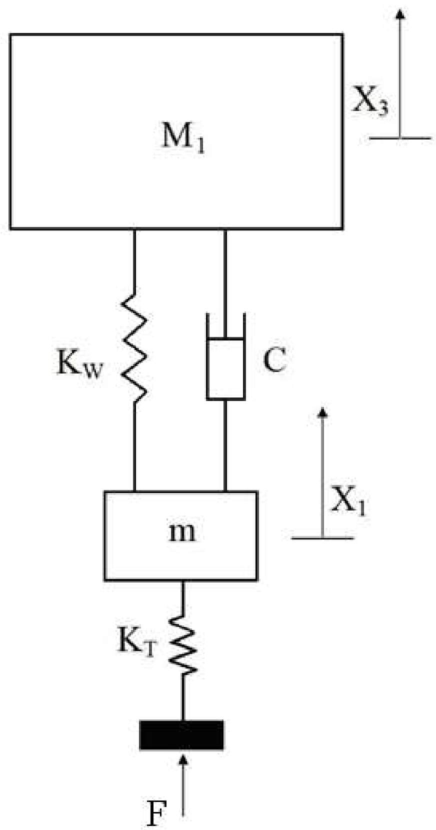


Figure 2.4: Quarter Car Model

Where

m=unsprung mass

M_1=sprung mass at one wheel

X_1=vertical displacement of the unsprung mass

X_3=vertical displacement of the sprung mass

K_W=wheel rate

K_T=tyre rate

c=wheel damping rate

The Quarter Car Model assumes that the road is a sine wave with amplitude A_1. Therefore, the excitation caused by the road is

$$F = A_1 e^{j\omega t} \tag{2.1}$$

Assumptions of the Quarter Car Model

Listed below are the assumptions incorporated into the quarter car model

- The CG of the sprung mass is located directly above the CG of the unsprung mass
- The quarter of the vehicle being analysed acts independently to the rest of the vehicle
- There is no tyre damping
- The damping of the wheel is theoretical linear fluid damping
- The excitation from the road is sinusoidal excitation with amplitude A_1

The Quarter Car Model will be used to determine the vertical accelerations of the sprung and unsprung masses. Please refer to Appendix C for the Quarter Car Model vertical acceleration calculations.

Half Car Model

The Half Car Model, sometimes called the bicycle model, is used to simulate one half of the car. It is more realistic than the quarter car model, because the front and rear are coupled so that their motions affect one another. It has four degrees of freedom: the vertical movement of the front unsprung mass, the vertical movement of the rear unsprung mass, the vertical movement of the centre of gravity of the sprung mass, and the pitching of the sprung mass about its CG.

Below is a diagram of the Half Car Model.

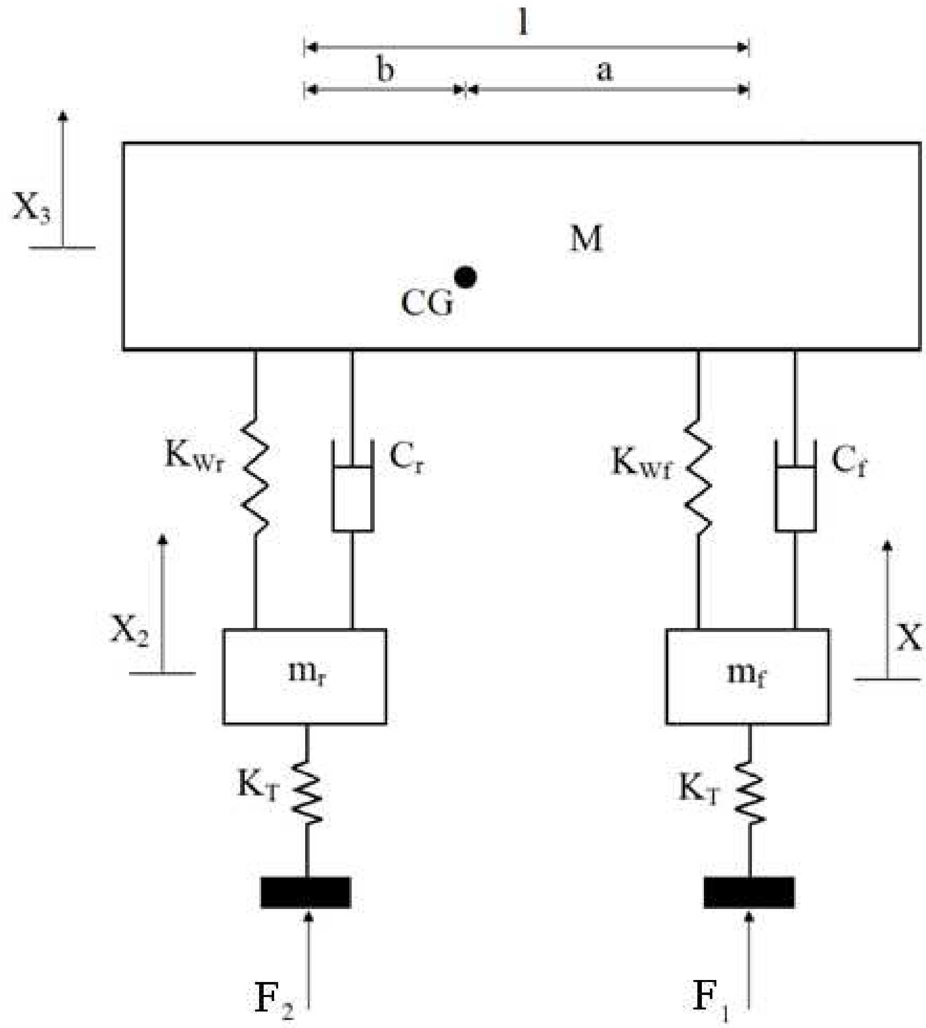


Figure 2.5: Half Car Model

Where

m_f=front unsprung mass

m_r=rear unsprung mass

M=sprung mass of half of the car

X_1=vertical displacement of the front unsprung mass

X_2=vertical displacement of the rear unsprung mass

X_3=vertical displacement of the CG of the sprung mass

K_{Wf}=front wheel rate

K_{Wr}=rear wheel rate

K_T=tyre rate

c_f=front wheel damping rate

c_r=rear wheel damping rate

l=wheelbase

a=distance from CG to front track

b=distance from CG to rear track

The Half Car Model assumes that the road is a sine wave with amplitude A_1, and with a length of one cycle of road profile of s. Below is a diagram of the assumed road.

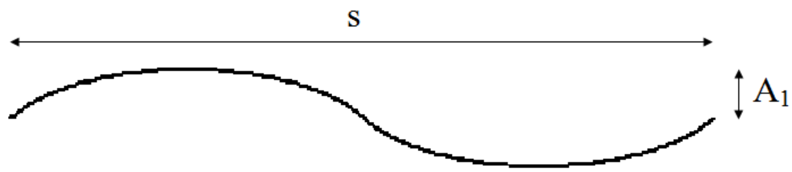


Figure 2.6: Assumed road

Therefore, the excitations caused by the road are:

$$F_1 = A_1 e^{j\omega t} \tag{2.2a}$$

$$F_2 = A_1 j \exp\left(-\frac{j2\pi l}{s}\right) e^{j\omega t} \tag{2.2b}$$

Please refer to Appendix D to see how the above excitations were calculated.

Below is a diagram of the sprung mass when it is at a pitch angle of θ. Not that the situation in the diagram occurs when the force from the front unsprung mass is greater than the force from the rear unsprung mass.

i.e.

$$K_{W_f}(X_1 - Y_1) + c_f(\dot{X}_1 - \dot{Y}_1) \; > \; K_{W_r}(X_2 - Y_2) + c_r(\dot{X}_2 - \dot{Y}_2) \tag{2.3}$$

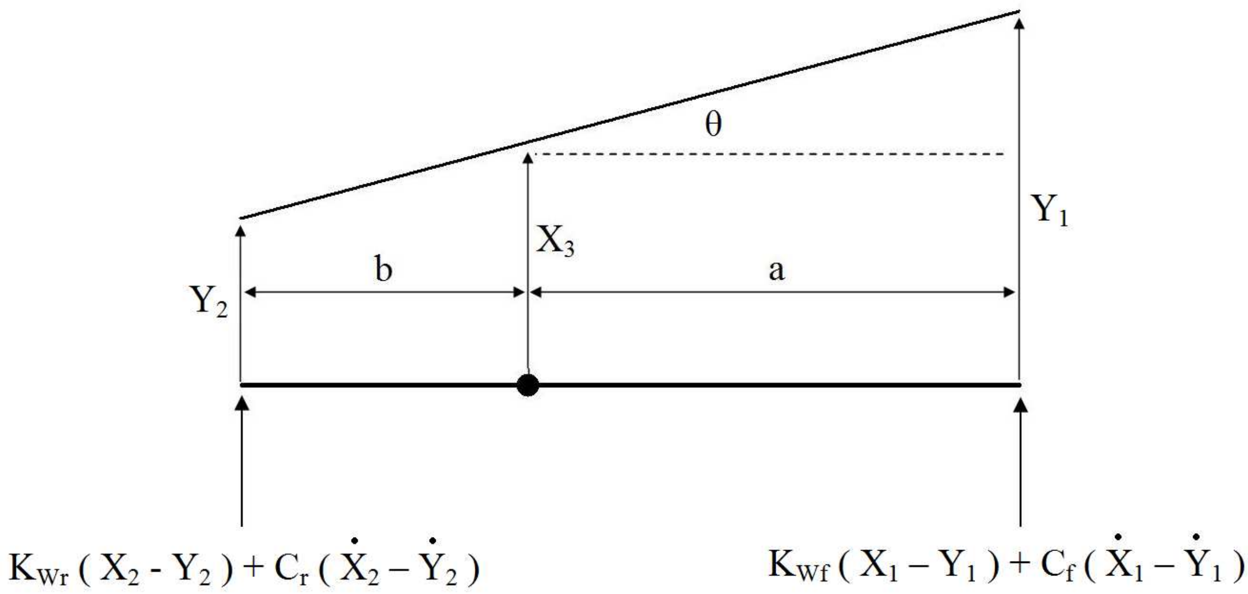


Figure 2.7: Sprung mass from the Half Car Model pitched at an angle θ.

Where, for small angles of θ,

$$Y_1 = X_3 + a\theta \tag{2.4a}$$

$$Y_2 = X_3 - b\theta \tag{2.4b}$$

Assumptions of the Half Car Model

Listed below are the assumptions incorporated into the half car model

- The car is travelling in a straight line
- The half of the vehicle that is being analysed acts independently to the rest of the vehicle
- There is no tyre damping
- The damping of the wheels is theoretical linear fluid damping
- The road is a sine wave with amplitude A_1 and with a length of one cycle of road profile of s
- The pitch angle, θ, is small
- The mass moment of inertia, I, is constant. This can be assumed as the mass moment of inertia will vary very little for small pitch angles.

The Half Car Model will be used to determine the vertical accelerations of the front unsprung, rear unsprung and sprung masses. Please refer to Appendix E for the Half Car Model vertical acceleration calculations.

3. Results

Available springs

The springs available to the REV team are an assortment of Fox coil springs, varying in spring rate (K_S) and travel distance. The spring rate and travel distance are written on the side of the spring. For example, 600 x 2.00 indicates a spring rate of 600 lbs/in and a travel distance of 2.00 in.

The table below lists the available Fox coil springs, as well as the accompanying spring rates in SI units.

Spring	**Spring Rate (N/m)**
600 x 2.00	105076
550 x 2.00	96320
500 x 2.30	87563
450 x 2.39	78807
300 x 2.57	52538

Table 3.1: Spring rates of the available Fox coil springs.

Motion ratio

If springs were mounted directly above the tyres, as pictured below, then the wheel rate would be equal to the spring rate.

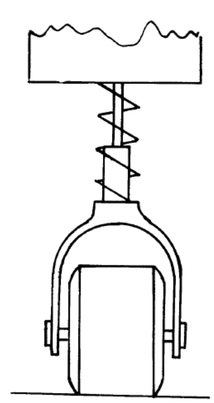


Figure 3.1: Spring mounted directly above wheel (Smith 1978)

However, due to limitations of space, it is necessary for the springs to be mounted inboard of the tyre centerline. This means that the wheel travel will not be equal spring travel. The motion ratio is defined as

$$\text{Motion Ratio} = \frac{\text{Wheel Travel}}{\text{Spring Travel}} \tag{3.1}$$

For example, in the diagram below, the motion ratio is 2.2 (Smith 1978).

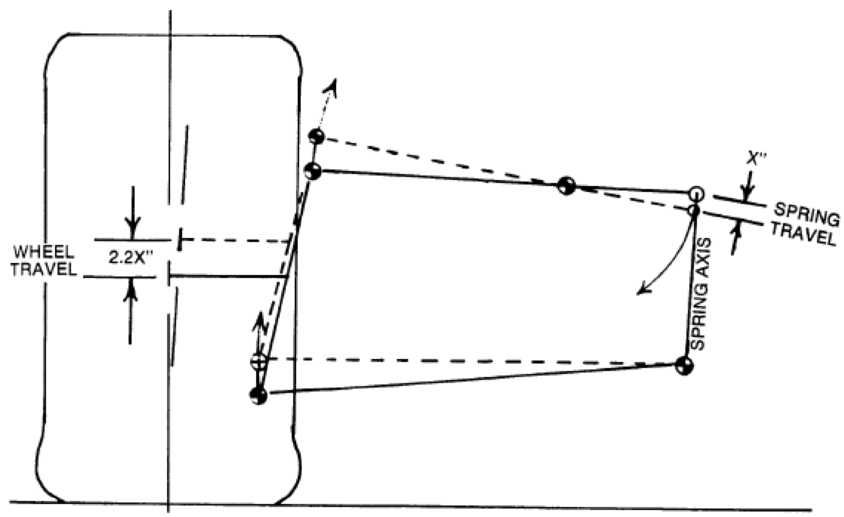


Figure 3.2: Spring mounted inboard of the tyre centerline (Smith 1978)

Using experimental data from the 2001 Motorsport team, the motion ratio for the front and rear wheels was calculated to be 1.764.

<u>Wheel rates</u>

The wheel rate (K_W) is defined as

$$K_W = \frac{K_S}{(\text{Motion Ratio})^2} \tag{3.2}$$

The table below shows the available springs and the corresponding wheel rates.

Spring	Wheel Rate (N/m)
600 x 2.00	33768
550 x 2.00	30954
500 x 2.30	28140
450 x 2.39	25326
300 x 2.57	16884

Table 3.2: Available springs and the corresponding wheel rates.

Tyre rate

A standard Goodyear racing tyre suitable for the application of the REV car with an inflation pressure of 83kPa has a tyre rate (K_T) of 180190N/m (Winzer 2002). This value was used for further calculations involving both the front and rear tyre rates.

Ride rates

The ride rate (K_R) is the combined rate of the wheel rate and the tyre rate. The deflections of the wheel and the tyre are additive, so therefore the formula for two springs in series is used (Milliken & Milliken 2002). The formula for ride rate is

$$\frac{1}{K_R} = \frac{1}{K_T} + \frac{1}{K_W} \tag{3.3a}$$

or

$$K_R = \frac{K_T K_W}{K_T + K_W} \tag{3.3b}$$

The table below shows the available springs and the corresponding ride rates.

Spring	Ride Rate (N/m)
600 x 2.00	28439
550 x 2.00	26416
500 x 2.30	24339

450 x 2.39	22205
300 x 2.57	15437

Table 3.3: Available springs and the corresponding ride rates

Undamped natural frequencies of the front and rear sprung masses

The natural frequency of the front sprung mass is

$$f_{Nf} = \frac{1}{2\pi}\sqrt{\frac{2K_R}{M_f}} \tag{3.4a}$$

The natural frequency of the rear sprung mass is

$$f_{Nr} = \frac{1}{2\pi}\sqrt{\frac{2K_R}{M_r}} \tag{3.4b}$$

At each end of the car there are two wheels, meaning that at each end there are two "ride rate" springs. These springs share a load, and are said to be in parallel. Therefore to find their composite rate, they are added. This explains the "$2K_R$" term in both of the equations above (Milliken & Milliken 1995).

The front and rear sprung masses were calculated to be 97.29kg and 104.71kg respectively.

The table below shows the available springs and the corresponding undamped natural frequencies of the front and rear sprung masses.

Spring	**Natural Frequency of Front Sprung Mass (Hz)**	**Natural Frequency of Rear Sprung Mass (Hz)**
600 x 2.00	3.848	3.709
550 x 2.00	3.709	3.575
500 x 2.30	3.560	3.432
450 x 2.39	3.400	3.278
300 x 2.57	2.835	2.733

Table 3.4: Available springs and the corresponding natural frequencies of the front and rear sprung masses

Spring selection

To select which combination of springs to use at the front and the rear of the car, two different criteria had to be satisfied. Any combination that satisfied both criteria was then considered a possible candidate for use in the REV car suspension.

Criteria1

Staniforth recommends that, as a general guideline, the frequency of the front sprung mass should be about 10% lower than the frequency of the rear sprung mass, in order to avoid the front and rear of the vehicle oscillating violently out of phase after going over a bump (Staniforth 1999).

This means that

$$\frac{f_{Nf}}{f_{Nr}} \approx 0.90 \qquad (3.5)$$

However, he states that his race experience has shown that this guideline is more of a "rule of thumb", and is not always valid. Therefore, the author has been rather lenient and considered any spring combination which produces frequency ratios in the range of

$$0.75 \le \frac{f_{Nf}}{f_{Nr}} \le 0.95 \qquad (3.6)$$

to have satisfied Criteria 1.

Below is a table showing the values of f_{Nf} / f_{Nr} for each spring combination. The combinations satisfying Criteria 1 are highlighted in bold.

Rear Springs	**f_{Nr} (Hz)**	**Front Springs**	**f_{Nf} (Hz)**	**f_{Nf} / f_{Nr}**
600 x 2.00	3.709	600 x 2.00	3.848	1.037
600 x 2.00	3.709	550 x 2.00	3.709	1
600 x 2.00	3.709	500 x 2.30	3.56	0.960
600 x 2.00	**3.709**	**450 x 2.39**	**3.4**	**0.917**
600 x 2.00	**3.709**	**300 x 2.57**	**2.835**	**0.764**
550 x 2.00	3.575	550 x 2.00	3.709	1.037
550 x 2.00	3.575	500 x 2.30	3.56	0.996
550 x 2.00	3.575	450 x 2.39	3.4	0.951
550 x 2.00	**3.575**	**300 x 2.57**	**2.835**	**0.793**
500 x 2.30	3.432	500 x 2.30	3.56	1.037
500 x 2.30	3.432	450 x 2.39	3.4	0.991
500 x 2.30	**3.432**	**300 x 2.57**	**2.835**	**0.826**
450 x 2.39	3.278	450 x 2.39	3.4	1.037
450 x 2.39	**3.278**	**300 x 2.57**	**2.835**	**0.865**
300 x 2.57	2.733	300 x 2.57	2.835	1.037

Table 3.5: Criteria 1

Criteria 2

In his notes, Maurice Olley suggests that the natural frequencies of the front and rear sprung masses should be close, however, the natural frequency of the rear sprung mass should be no greater than 1.2 times the natural frequency of the front sprung mass (Milliken & Milliken 2002).

This means that

$$f_{Nr} \leq 1.2 f_{Nf} \qquad (3.7)$$

As in Criteria 1, this is to avoid the situation where the front and rear of the vehicle oscillate violently out of phase after going over a bump.

Below is a table showing the values of $1.2f_{Nf}$ for each spring combination. The combinations satisfying Criteria 2 are highlighted in bold.

Rear Springs	**f_{Nr} (Hz)**	**Front Springs**	**f_{Nf} (Hz)**	**1.2 f_{Nf}**
600 x 2.00	**3.709**	**600 x 2.00**	**3.848**	**4.6176**
600 x 2.00	**3.709**	**550 x 2.00**	**3.709**	**4.4508**
600 x 2.00	**3.709**	**500 x 2.30**	**3.56**	**4.272**
600 x 2.00	**3.709**	**450 x 2.39**	**3.4**	**4.08**
600 x 2.00	3.709	300 x 2.57	2.835	3.402
550 x 2.00	**3.575**	**550 x 2.00**	**3.709**	**4.4508**
550 x 2.00	**3.575**	**500 x 2.30**	**3.56**	**4.272**
550 x 2.00	**3.575**	**450 x 2.39**	**3.4**	**4.08**
550 x 2.00	3.575	300 x 2.57	2.835	3.402
500 x 2.30	**3.432**	**500 x 2.30**	**3.56**	**4.272**
500 x 2.30	**3.432**	**450 x 2.39**	**3.4**	**4.08**
500 x 2.30	3.432	300 x 2.57	2.835	3.402
450 x 2.39	**3.278**	**450 x 2.39**	**3.4**	**4.08**
450 x 2.39	**3.278**	**300 x 2.57**	**2.835**	**3.402**
300 x 2.57	**2.733**	**300 x 2.57**	**2.835**	**3.402**

Table 3.6: Criteria 2

Spring combination candidates

Therefore, the combinations of springs satisfying both criteria are

Combination	**Front Springs**	**Rear Springs**
1	450 x 2.39	600 x 2.00
2	300 x 2.57	450 x 2.39

Table 3.7: Candidates for the spring combination of the car

Wheel hop: rate and mass ratios

Assuming that the oscillation centres of the front and rear sprung masses are directly above the front and rear tracks respectively,

Critical damping of the sprung mass is

$$c_{crit,M} = 2\sqrt{M_1 \frac{K_T K_W}{K_T + K_W}} \tag{3.8}$$

Critical damping of the unsprung mass is

$$c_{crit,m} = 2\sqrt{m(K_T + K_W)} \tag{3.9}$$

Where,

M_1=sprung mass at one wheel
m=unsprung mass

For wheel hop and ride to be equally damped,

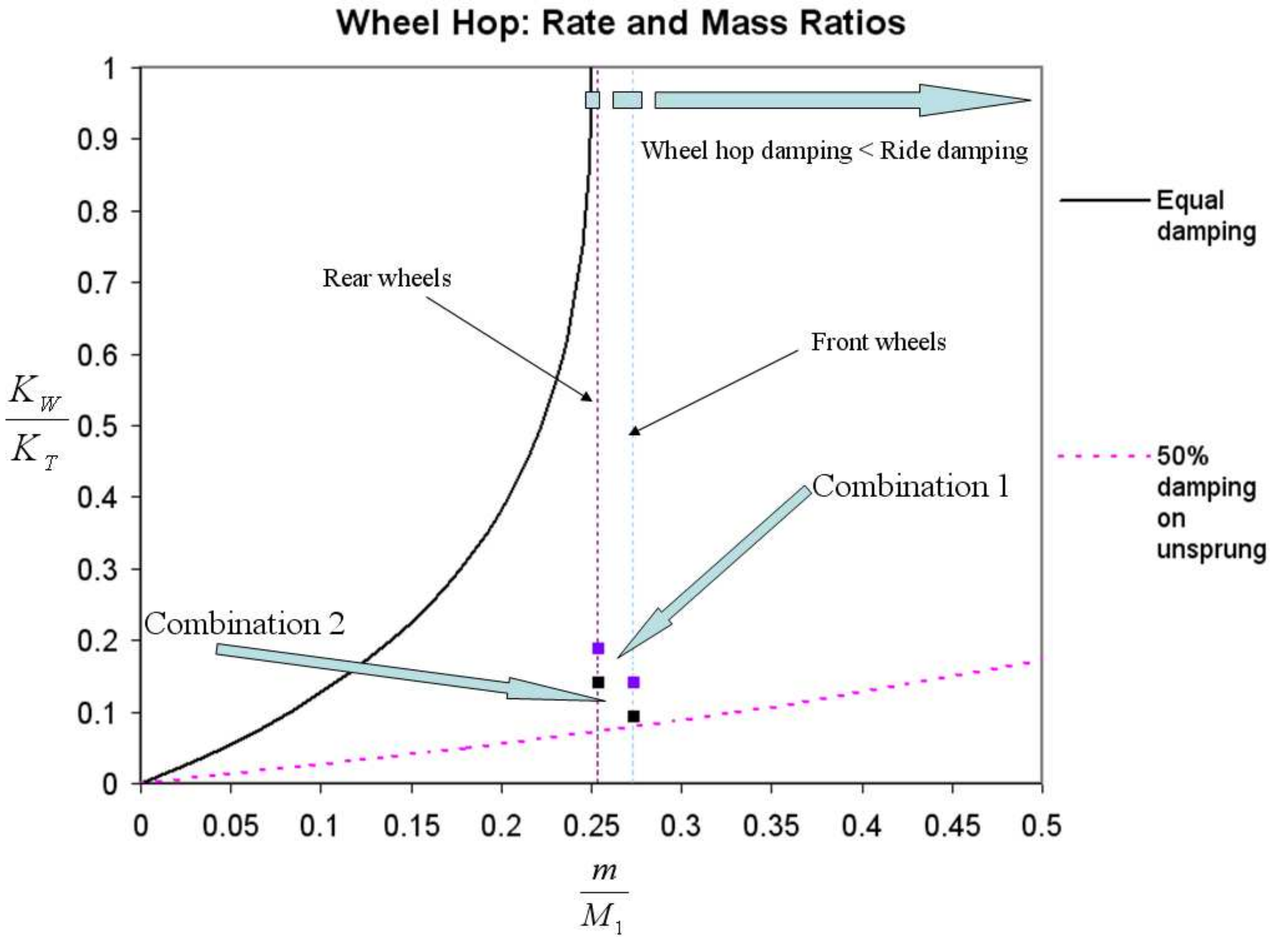
$$c_{crit,M} = c_{crit,m} \tag{3.10a}$$

This can be rearranged to give

$$\frac{m}{M_1} = \frac{\dfrac{K_W}{K_T}}{\left(1 + \dfrac{K_W}{K_T}\right)^2} \tag{3.10b}$$

(Milliken & Milliken 2002)

Below is a graph of K_W/K_T vs. m/M_1



Graph 3.1: Wheel Hop: Rate and Mass Ratios

Both spring combination candidates have been plotted onto the graph. For both spring combinations, the mass ratios are the same.

For the front wheels,

$$\frac{m}{M_1} \approx 0.27$$

For the rear wheels,

$$\frac{m}{M_1} \approx 0.25$$

The rate ratios differ between the spring combinations.

Combination 1

The rate ratio for the front wheels is

$$\frac{K_W}{K_T} \approx 0.14$$

The rate ratio for the rear wheels is

$$\frac{K_W}{K_T} \approx 0.19$$

Combination 2

The rate ratio for the front wheels is

$$\frac{K_W}{K_T} \approx 0.09$$

The rate ratio for the rear wheels is

$$\frac{K_W}{K_T} \approx 0.14$$

Anywhere to the right of the "equal damping" line means that wheel hop damping is less than ride damping. This means that in order to damp out wheel hop, the ride will be overdamped. As can been seen from the graph, the plotted points (especially the front wheel from Combination 2) are closer to "50% damping on unsprung" than to "equal damping".

To get closer the "equal damping" line, and therefore not to have to overdampen ride to prevent wheel hop, there are a variety of changes that can be made.

I. The unsprung mass could be reduced. This could prove difficult, but even manufacturing the wishbones out of carbon fibre instead of mild steel would help improve the situation.

II. The sprung mass could be increased. An acceptable way of doing this would be to modify the battery cage to house a larger number of lithium-ion batteries. Although this would be expensive, it would not only increase the sprung mass, but it would also allow the car to travel a greater distance before having to recharge the batteries. Another way would be to put some panels and a body kit on the car, as the chassis is currently just a “skeleton” frame, as can be seen on the screen shot of the Solid Works model of the car in Figure 2.1.

III. Stiffer springs could be used. This would be a simple change to make, however, if the springs are too stiff, the ride will be too uncomfortable for the driver. For further information, please refer to “The problem associated with using springs that are too stiff”.

<u>Vertical accelerations produced by the spring combination candidates</u>

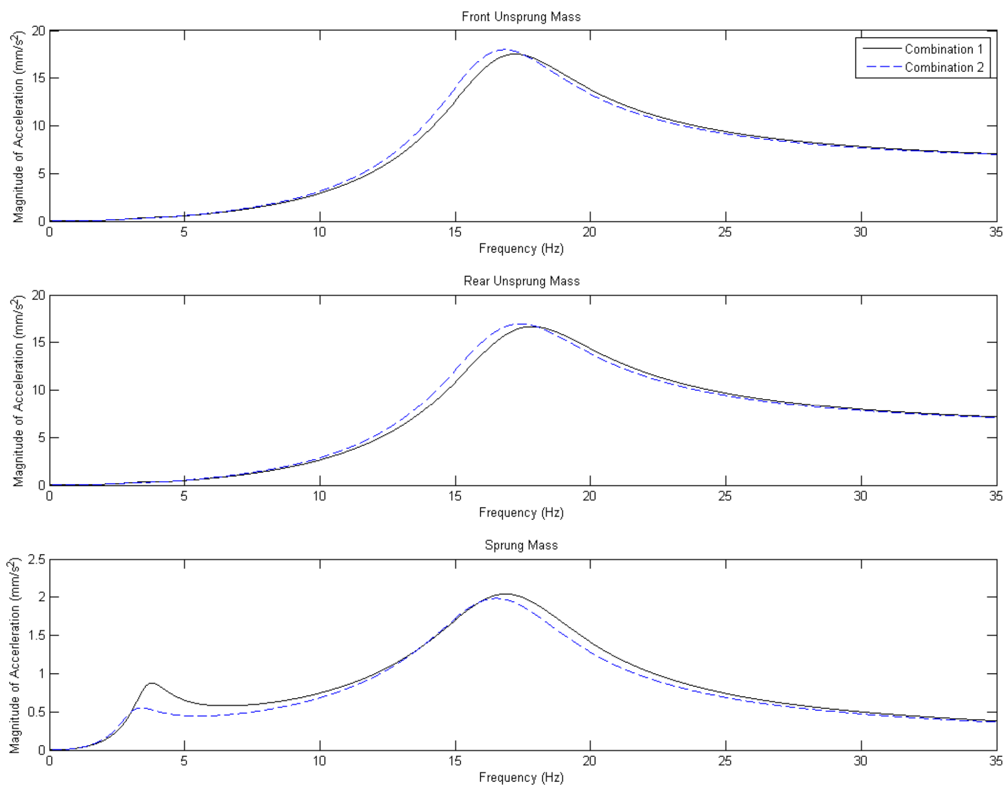
Both candidates for the spring combination were entered into the Half Car Model.

The “road” parameters were arbitrarily chosen as follows.

A_1 = 0.1m
s = 10m

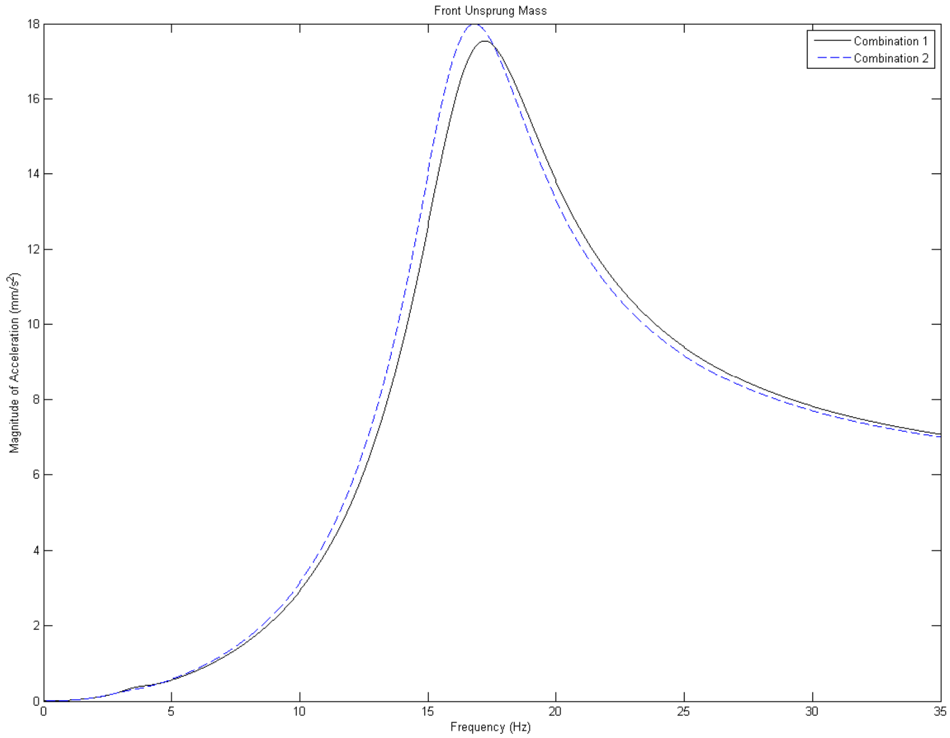
The wheel damping rates were arbitrarily chosen to be 15% of the critical damping of the unsprung masses (damping ratio, ζ=0.15). Please refer to Appendix F for the calculation of the damping rates.

Below are the MATLAB generated graphs of magnitude of vertical acceleration vs. frequency of the two unsprung masses and the sprung mass from the Half Car Model using both combinations of springs. Please refer Appendix G for the MATLAB code for the Half Car Model vertical accelerations.



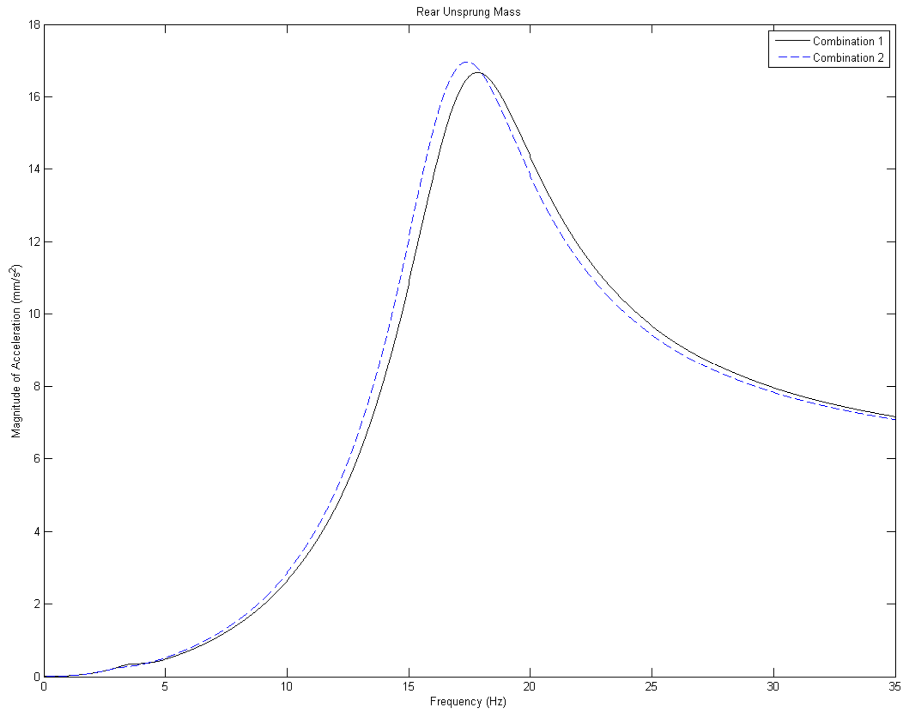
Graph 3.2: Vertical accelerations of the three masses at frequencies between 0-35Hz.

Below is a MATLAB generated graph showing the magnitude of vertical acceleration vs. frequency of the front unsprung mass from the Half Car Model using both combinations of springs.



Graph 3.3: Vertical acceleration of the front unsprung mass at frequencies between 0-35Hz

Below is a MATLAB generated graph showing the magnitude of vertical acceleration vs. frequency of the rear unsprung mass from the Half Car Model using both combinations of springs.

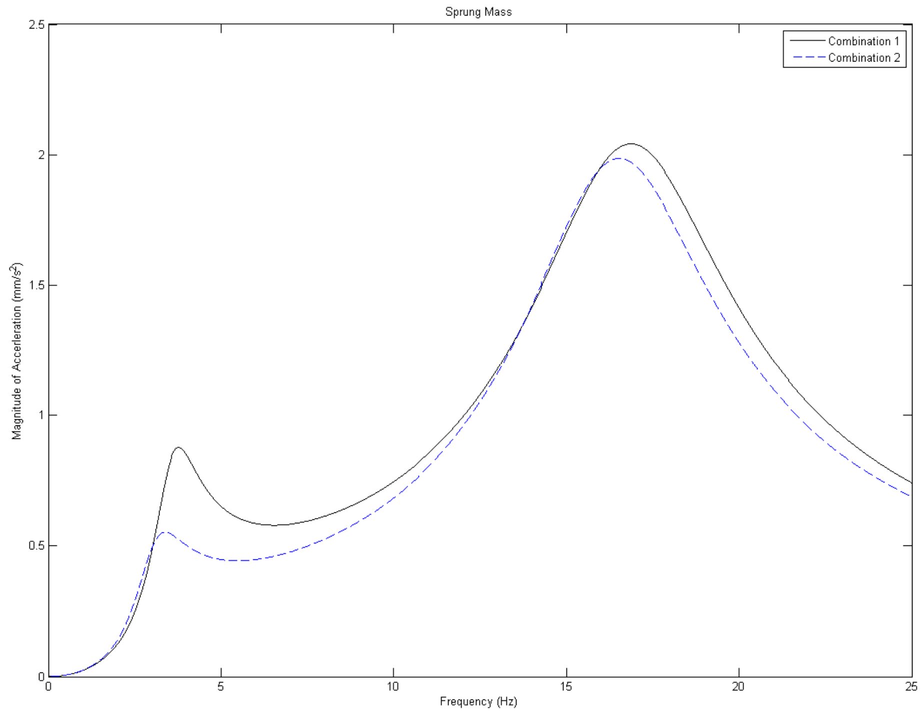


Graph 3.4: Vertical acceleration of the rear unsprung mass at frequencies between 0-35Hz

As can be seen from the acceleration-frequency graphs of the front and rear unsprung masses, Combination 2 produces slightly larger vertical acceleration of the unsprung masses compared to Combination 1, although this difference is so small that it can be considered negligible.

Vehicle suspension is used to minimise discomfort cause by irregularities in the road. A common method of determining the extent of this discomfort is to measure the vertical acceleration experienced by the driver, i.e. the vertical acceleration of the sprung mass (Milliken & Milliken 1995).

Below is a MATLAB generated graph showing a "close up" of the magnitude of vertical acceleration vs. frequency of the sprung mass from the Half Car Model using both combinations of springs.



Graph 3.5: Vertical acceleration of the sprung mass at frequencies between 0-25Hz

On closer inspection of the acceleration-frequency graph of the sprung mass, it can be seen that Combination 2 produces lower vertical accelerations compared to Combination 1. This means that Combination 2 would provide a more comfortable ride for the driver, and, therefore would be the most appropriate combination of springs to be used on the REV car.

Most appropriate springs to be used on the REV car

So the most appropriate springs to be used on the REV car are:

	Rear Springs	**Front Springs**
Combination 2	450 x 2.39	300 x 2.57

Table 3.8: Most appropriate springs to be used on the REV car

The problem associated with using springs that are too stiff

Referring to the "Wheel Hop: Rate and Mass Ratios" graph, one of the suggestions to get closer to the "equal damping" line was to use stiffer springs. The problem associated with using springs that are too stiff is that the ride can become too uncomfortable for the driver.

This can be shown by the following extreme example.

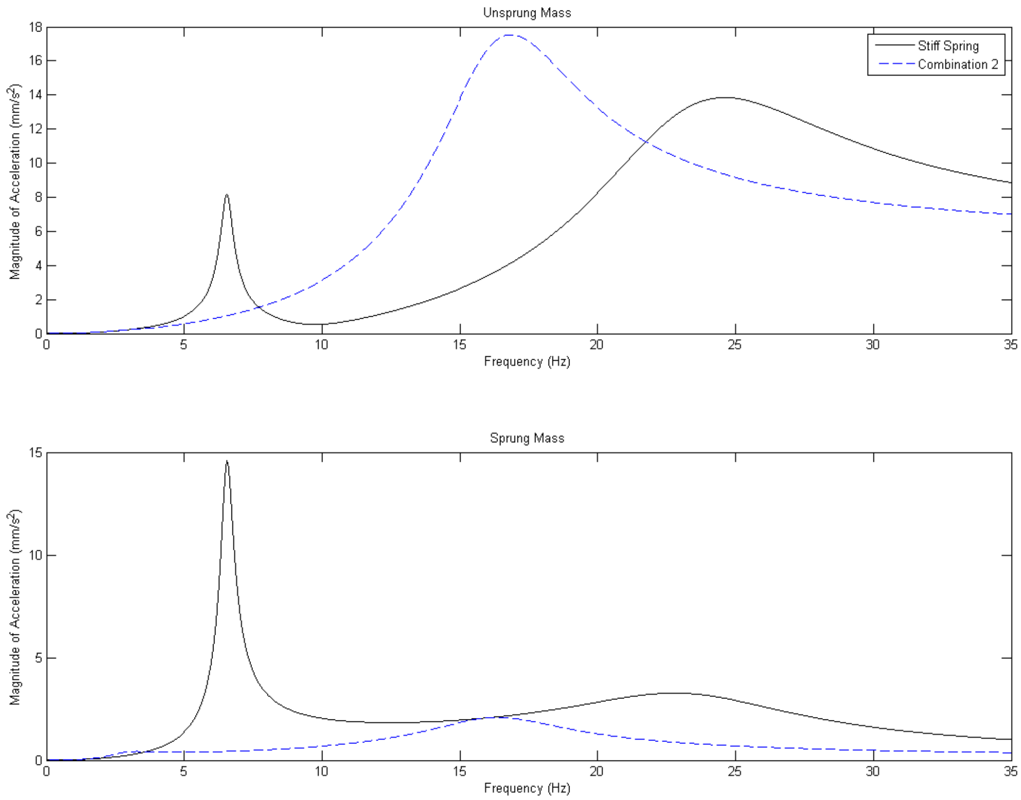
On the "Wheel Hop: Rate and Mass Ratios" graph, the plotted points have an m/M_1 ratio of either 0.25 or close to 0.25. On the "equal damping" line, an m/M_1 ratio of 0.25 corresponds to a K_W/K_T ratio of 1.

This means that in order to be on the "equal damping" line, the wheel rate must be the same as the tyre rate, which is 180 190 N/m. Taking into account the motion ratio, this corresponds to a spring rate of just over 3200lbs/in.

Both the new "stiff spring" (where the wheel rate is equal to the tyre rate) and the springs from Combination 2 were entered into the Quarter Car Model.

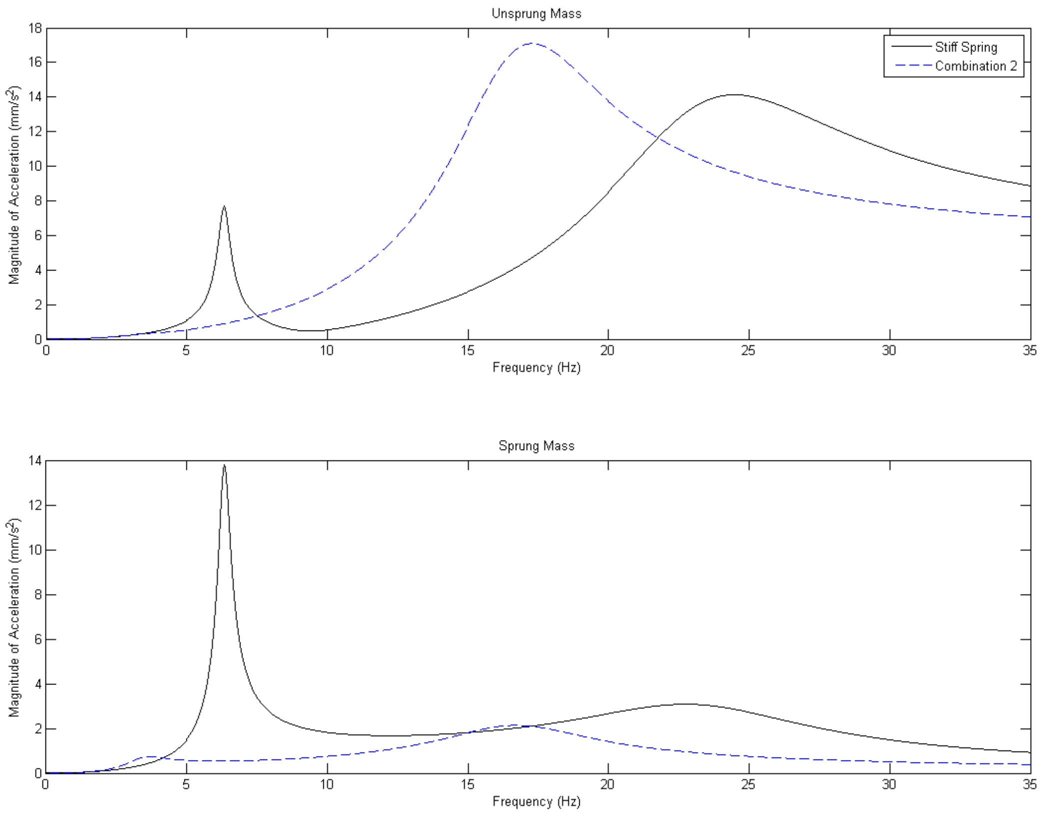
Arbitrarily, the "road" amplitude A_1 was chosen to be 0.1m, and the wheel damping rates were chosen to be 15% of the critical damping of the unsprung masses (damping ratio, ζ=0.15). Please refer to Appendix H for the calculation of the damping rates.

Below are the MATLAB generated graphs of magnitude of vertical acceleration vs. frequency of the unsprung and the sprung masses of the front wheel from the Quarter Car Model using both the "stiff spring" and the front spring from Combination 2. Please refer Appendix I for the MATLAB code for the Quarter Car Model vertical accelerations.



Graph 3.6: Vertical accelerations of the sprung and unsprung masses of the front wheel at frequencies between 0-35Hz

Below are the MATLAB generated graphs of magnitude of vertical acceleration vs. frequency of the unsprung and the sprung masses of the rear wheel from the Quarter Car Model using both the “stiff spring” and the rear spring from Combination 2.

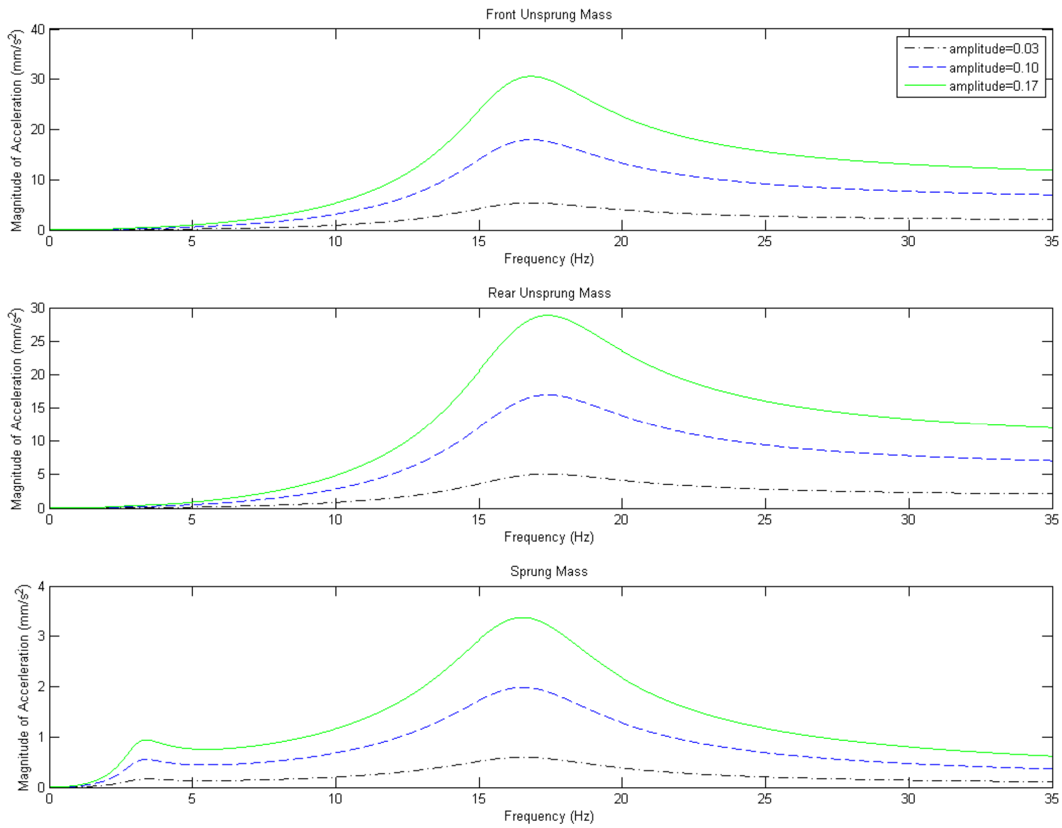


Graph 3.7: Vertical accelerations of the sprung and unsprung masses of the rear wheel at frequencies between 0-35Hz

As can be observed from the two previous graphs, the "stiff spring" produces very large "spikes" in the vertical accelerations of the sprung masses of the front and rear wheels. This is a clear indication that using springs this stiff would cause the driver to be very uncomfortable.

<u>Changing amplitude, A_1</u>

Below are the MATLAB generated graphs of magnitude of vertical acceleration vs. frequency of the two unsprung masses and the sprung mass from the Half Car Model using the Combination 2 springs. These graphs show the effect of changing the "road" amplitude A_1.



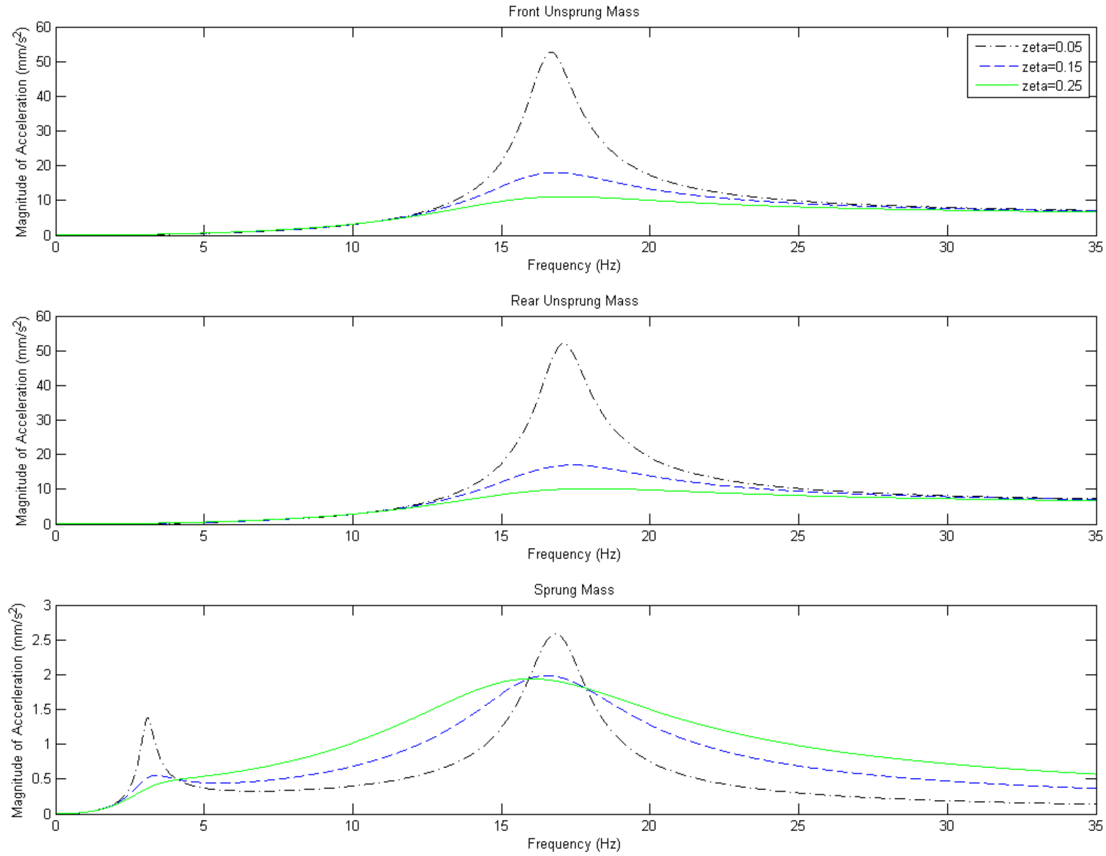
Graph 3.8: Changing the amplitude

The MATLAB graphs shows curves for A_1=0.03m, A_1=0.1m and A_1=0.17m.

It is clear from observing the graphs that increasing the amplitude of the "road" causes the vertical accelerations of each of the masses to increase.

<u>Changing damping ratio, ζ</u>

Below are the MATLAB generated graphs of magnitude of vertical acceleration vs. frequency of the two unsprung masses and the sprung mass from the Half Car Model using the Combination 2 springs. These graphs show the effect of changing the damping ratio ζ.



Graph 3.9: Changing the damping ratio

The MATLAB graphs shows curves for ζ=0.05, ζ =0.15 and ζ =0.25.

In the case of the front and rear unsprung masses, increasing the damping ratio reduces the vertical accelerations at all frequencies.

However, in the case of the sprung mass, increasing the damping ratio reduces the acceleration peaks at around 3Hz and 17Hz, but causes the acceleration level over the remainder of the frequency range to be increased.

Using a damping ratio of 0.15 provides a reasonable compromise of reducing the unsprung mass acceleration peak, while reducing both sprung mass acceleration peaks, and producing sprung mass acceleration levels that are not too high for the remainder of the frequency range.

4. Safety

It was vitally important that high levels of safety were adhered to during this project, to ensure the wellbeing of the author and other members of the University, including other students, lecturers, tutors and lab technicians. It is responsibility of everybody associated with the REV project to assist in creating and maintaining a culture of safety, encouraging all participants to remain vigilant against possible health and safety hazards.

Emergency phone numbers

In the case of any emergency, call 2222 (from any internal university phone) or 6488 2222 (from a mobile phone), and give the following information:

- type of emergency
- location
- name
- mobile phone number or the telephone extension of the room

Below is a list of some other important phone numbers

Hospital (QE 2 'G Block' Emergency)	(0) 9346 3380
Doctor (UWA Medical Centre)	Extension 2118
Poisons Information Centre (All hours)	0 13 11 26

Fire procedure

In the event of a fire, smoke detectors should sound an alarm. If the fire is too large to extinguish, or the person or persons are not comfortable with fighting the fire, they should immediately evacuate the area, and assemble at the designated evacuation area. If the fire is small and the person or persons are comfortable with fighting the fire, they should select an appropriate fire extinguisher and extinguish the fire from a safe distance. Most fire extinguishers are the carbon dioxide or dry chemical type, and they are suitable for most fire. Beware that foam and water extinguishers are not to be used on electrical fires, and water extinguishers are not to be used on flammable liquid fires.

Smoking

To protect people from tobacco smoke, smoking is strictly forbidden within buildings or within 5 metres from the entrance to a building. Also, smoking is not permitted in the vicinity of gas storage bottles.

Electrical safety

It is important that all electrical equipment is properly used and well maintained. All equipment should be regularly inspected, and any equipment believed to be unsafe or faulty should be clearly tagged, and not used until it has been fixed and approved by a qualified electrician.

Working in labs

When working in the labs, personal protective equipment (PPE), such as fully enclosed footwear and approved safety glasses, must be worn at all times. When working alone outside of the normal operating hours of 9am to 5pm, people are not permitted to use tools.

Driving the REV car

When driving the REV car, fully enclosed footwear and a helmet must be worn. The driver must also be strapped in the seat with a five-point safety harness.

If when the car is in motion, one or more of the wheels starts to oscillate violently due to wheel hop or some other form of unsprung mass oscillation, it is important not to immediately brake. This will cause the frequency of vibration to decrease, while the amplitude will increase significantly, which can lead to loss of control of the car. Instead, it is recommended that the driver simply takes their foot off the accelerator and allows the car to slow down until the violent oscillation of the wheel(s) stops. The car suspension should then be immediately serviced, meaning that the dampers should be either re-tuned or replaced (if damaged) so that they are able to damp out any violent oscillations of the wheels.

5. Conclusion and Future Work

After considering both spring selection criteria and observing the acceleration-frequency graphs generated by MATLAB, the most suitable spring combination to use on the REV car is the 300 x 2.57 springs at the front and the 450 x 2.39 springs at the rear (Combination 2). As in Combination 1, these springs satisfied both criteria and produced very similar vertical accelerations for both the front and rear unsprung masses, however, they produced much smaller vertical accelerations of the sprung mass compared to Combination 1, meaning that they would provide a much more comfortable ride for the driver.

After observing the "Wheel Hop: Rate and Mass Ratios" graph, it became apparent that there are three methods in order get closer to the "equal damping" line, and to avoid overdamping the ride to control wheel hop. The first method is to reduce the unsprung mass. This could be achieved by replacing the current mild steel wishbones with ones that are manufactured out of carbon fibre. Another way to reduce unsprung mass would be to redesign the uprights so that they are lighter, although doing this would possibly be a final year project by itself. The second method is to increase the sprung mass. As can be seen by the screen shot Solid Works model in Figure 2.1, the car chassis consists only of a frame, and is in "skeleton" form. By adding panels and a body it, the sprung mass would be increased, and the car would be more aesthetically pleasing and may even perform better aerodynamically. The third and last method was to use stiffer springs. This, however, should be done with caution, as using springs that are too stiff can have a detrimental effect on ride.

There is plenty of future research that can be done on the REV car. This project has only simulated situations where the car is pitching. It would be desirable if future REV students could model transient situations where the car is turning, and the associated rolling, yawing and pitching of the car.

Another interesting study would be to design and implement a spring-damper arrangement that could be attached to the base of the seat, which could damp out any violent oscillations of the sprung mass. This would allow stiffer springs to be used, in order to control wheel hop, and would ensure that the ride was comfortable for the driver.

If the REV car is going to compete in future competitions, it would be advantageous for a kinetic suspension system to be investigated. The UWA Motorsport team has been using this technology since 2004, and it seems as though using a kinetic suspension system will be the way of the future

6. References

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7. Appendices

Appendix A: Negative swing arm effect

The negative swing arm effect causes camber to become increasingly positive as the wheel rises, as shown below.

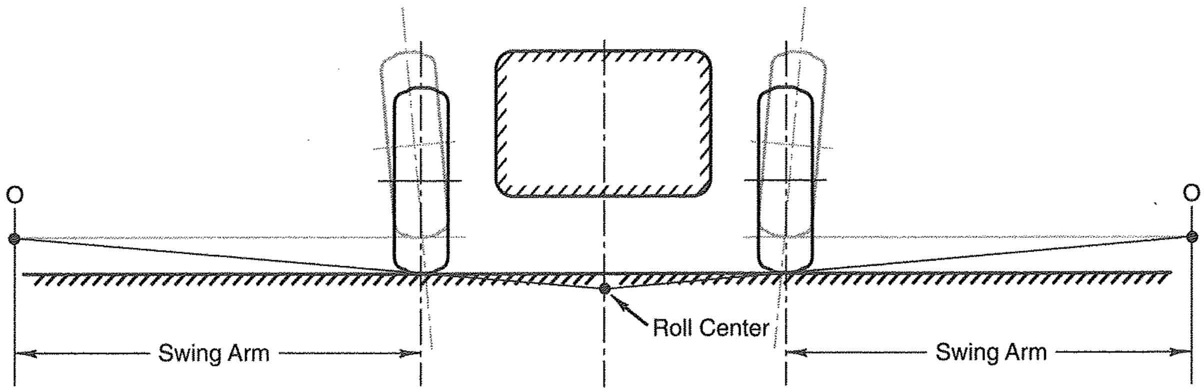

Figure A: Negative swing arm effect (Milliken & Milliken 2002)

Appendix B: Determining a and b

A set of scales was placed underneath each wheel of the car, which was “loaded” with a 75 kg driver.

The average scale readings of the front and rear wheels was 66kg and 71kg respectively.

This information can be represented in the diagram below.

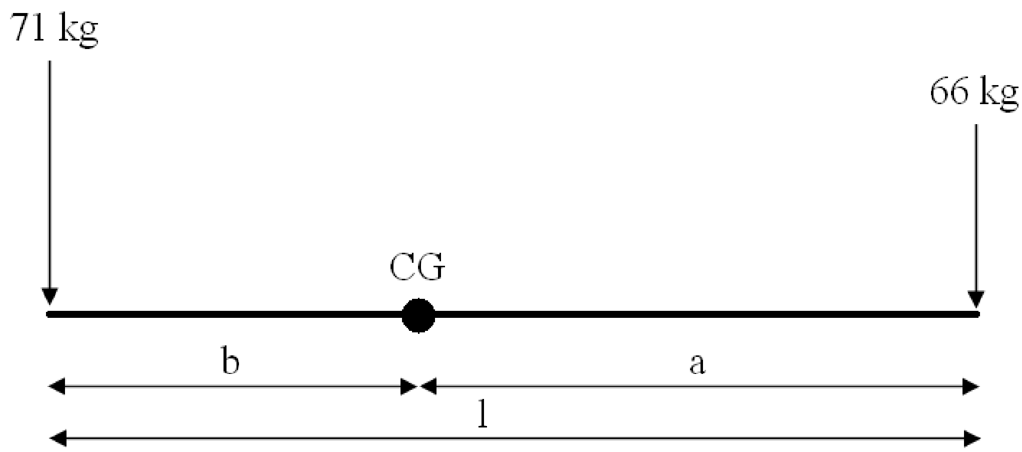

Figure B: Mass representation diagram

$$l = a + b = 1.85m$$

$$\sum M_{CG} = 0$$

$$\therefore 71b = 66a$$

$$71(l - a) = 66a$$

$$\therefore a = 0.959m$$

$$\therefore b = 0.891m$$

Appendix C: Quarter Car Model Vertical Acceleration Calculations

$$[MassMatrix] = \begin{bmatrix} m & 0 \\ 0 & M \end{bmatrix}$$

$$[DampingMatrix] = \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}$$

$$[SpringMatrix] = \begin{bmatrix} K_T + K_W & -K_W \\ -K_W & K_W \end{bmatrix}$$

$$\underline{X} = \begin{pmatrix} X_1 \\ X_3 \end{pmatrix}$$

$$\underline{X_0} = \begin{pmatrix} X_{1_0} \\ X_{3_0} \end{pmatrix}$$

$$\underline{X} = \underline{X_0} e^{j\omega t}$$

$$\underline{A} = \begin{pmatrix} A_1 \\ 0 \end{pmatrix}$$

$$[MassMatrix]\underline{\ddot{X}} + [DamperMatrix]\underline{\dot{X}} + [SpringMatrix]\underline{X} = \underline{A}e^{j\omega t}$$

$$-\omega^2 [MassMatrix]\underline{X_0} + j\omega[DampingMatrix]\underline{X_0} + [SpringMatrix]\underline{X_0} = \underline{A}$$

$$\left(-\omega^2[MassMatrix] + j\omega[DampingMatrix] + [SpringMatrix]\right)\underline{X_0} = \underline{A}$$

$$B = -\omega^2[MassMatrix] + j\omega[DampingMatrix] + [SpringMatrix]$$

$$\underline{X_0} = B^{-1}\underline{A}$$

$$\underline{X} = \underline{X_0}e^{j\omega t}$$

$$\underline{\dot{X}} = j\omega\underline{X_0}e^{j\omega t}$$

$$\underline{\ddot{X}} = -\omega^2\underline{X_0}e^{j\omega t}$$

Appendix D: Road Excitation Calculations

Assuming that road is a sine wave with amplitude A_1 and a length of one cycle of road profile of s, pictured below:

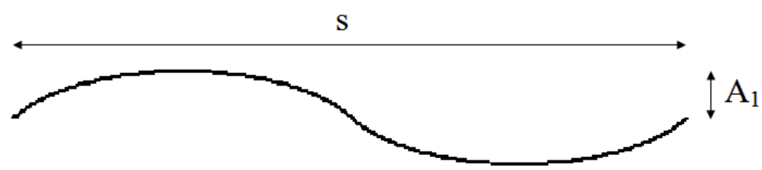

Figure D: Assumed road

Period of oscillation, T:

$$T = \frac{s}{v}$$

Where, s= length of one cycle of road profile

v= velocity

Frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{\frac{s}{v}} = \frac{2\pi v}{s}$$

Excitation at front wheel:

$$A_1 \sin \omega t$$

Converting to complex form:

$$A_1 e^{j\omega t}$$

Rear wheel will be excited some time after the front wheel, depending on the velocity of the vehicle.

Therefore, excitation at rear wheel:

$$
\begin{aligned}
&A_1 \sin\omega(t+\Delta t)\\
&= A_1 \sin\omega\left(t+\frac{l}{v}\right)\\
&= A_1 \sin\omega\left(t+\frac{2\pi l}{\omega s}\right)\\
&= A_1 \sin\left(\omega t+\frac{2\pi l}{s}\right)
\end{aligned}
$$

l = wheelbase

Converting to complex form:

$$
A_1\, j \exp\left(-\frac{j2\pi l}{s}\right) e^{j\omega t}
$$

Appendix E: Half Car Model Vertical Acceleration Calculations

$$MassMatrix = \begin{bmatrix} m_f & 0 & 0 & 0 \\ 0 & m_r & 0 & 0 \\ 0 & 0 & M & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

$$DamperMatrix = \begin{bmatrix} c_f & 0 & -c_f & 0 \\ 0 & c_r & -c_r & 0 \\ -c_f & -c_r & c_f + c_r & 0 \\ -ac_f & bc_r & ac_f - bc_r & a^2 c_f + b^2 c_r \end{bmatrix}$$

$$SpringMatrix = \begin{bmatrix} K_{T_f} + K_{W_f} & 0 & -K_{W_f} & 0 \\ 0 & K_{T_r} + K_{W_r} & -K_{W_r} & 0 \\ -K_{W_f} & -K_{W_r} & K_{W_f} + K_{W_r} & 0 \\ -aK_{W_f} & bK_{W_r} & aK_{W_f} - bK_{W_r} & a^2 K_{W_f} + b^2 K_{W_r} \end{bmatrix}$$

$$X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \theta \end{pmatrix}$$

$$X_0 = \begin{pmatrix} X_{1_0} \\ X_{2_0} \\ X_{3_0} \\ \theta_0 \end{pmatrix}$$

$$\underline{X} = \underline{X_0} e^{j\omega t}$$

$$\underline{A} = \begin{pmatrix} A_1 \\ A_1 j \exp\left(-\dfrac{j2\pi d}{s}\right) \\ 0 \\ 0 \end{pmatrix}$$

$$[MassMatrix]\underline{\ddot{X}} + [DamperMatrix]\underline{\dot{X}} + [SpringMatrix]\underline{X} = \underline{A} e^{j\omega t}$$

$$-\omega^2 [MassMatrix]\underline{X_0} + j\omega [DampingMatrix]\underline{X_0} + [SpringMatrix]\underline{X_0} = \underline{A}$$

$$\left(-\omega^2 [MassMatrix] + j\omega [DampingMatrix] + [SpringMatrix]\right)\underline{X_0} = \underline{A}$$

$$B = -\omega^2 [MassMatrix] + j\omega [DampingMatrix] + [SpringMatrix]$$

$$\underline{X_0} = B^{-1} \underline{A}$$

$$\underline{X} = \underline{X_0} e^{j\omega t}$$
$$\underline{\dot{X}} = j\omega \underline{X_0} e^{j\omega t}$$
$$\underline{\ddot{X}} = -\omega^2 \underline{X_0} e^{j\omega t}$$

Appendix F: Damping calculations for the Half Car Model

$$c = \zeta \times c_{crit}$$

$$c_{crit} = 2\sqrt{m(K_T + K_W)}$$

$$\zeta = 0.15$$

Damping of front unsprung mass:

Combination 1

c_{crit} = 3846.7 Ns/m

c = 0.15 x 3846.7 = 577.0 Ns/m

Combination 2

c_{crit} = 3766.9 Ns/m

c = 0.15 x 3766.9 = 565.0 Ns/m

Damping of rear unsprung mass:

Combination 1

c_{crit} = 3924.9 Ns/m

c = 0.15 x 3924.9 = 588.7 Ns/m

Combination 2

c_{crit} = 3846.7 Ns/m

c = 0.15 x 3846.7 = 577.0 Ns/m

Appendix G: MATLAB Code for Half Car Model Vertical Accelerations

Below is the MATLAB code for determining the vertical accelerations of the front unsprung, rear unsprung and sprung masses in the Half Car Model.

```
%Half Car Model - Acceleration
%Peter Corke 20139918

clear;

%Changeable variables

kwf=;           % front wheel spring rate [N/m]

kwr=;          % rear wheel spring rate [N/m]

cf=;             % front wheel damping rate [Ns/m]

cr=;            % rear wheel damping rate [Ns/m]

%Constant variables

mf=;            % front unsprung mass [kg]

mr=;             % rear unsprung mass [kg]

M=;            % sprung mass of half of the car[kg]

I=;              % mass moment of inertia of sprung mass [kg
m^2]

a=;             % distance from front wheel to CG [m]

b=;             % distance from rear wheel to CG [m]

l=a+b;            % wheelbase [m]

ktf=;         % front tyre spring rate [N/m]

ktr=;          % rear tyre spring rate [N/m}

A1=;             % Amplitude [m]

s=;               %length of one cycle of road profile [m]
```

```
hzmin=;         % Frequency range [Hz]
hzmax=;
n=1000;
hzamount=(hzmax-hzmin)/(n-1);


Mass_Matrix=[mf,0,0,0;0,mr,0,0;0,0,M,0;0,0,0,I];

Damper_Matrix=[cf,0,-cf,0;0,cr,-cr,0;-cf,-cr,cf+cr,0;-
a*cf,b*cr,a*cf-b*cr,a^2*cf+b^2*cr];

Spring_Matrix=[ktf+kwf,0,-kwf,0;0,ktr+kwr,-kwr,0;-kwf,-
kwr,kwf+kwr,0;-a*kwf,b*kwr,a*kwf-b*kwr,a^2*kwf+b^2*kwr];


A=[A1;A1*j*exp((-j*2*pi*l)/s);0;0];


wmin=hzmin*2*pi;
wmax=hzmax*2*pi;
wamount=hzamount*2*pi;
k=1;


for w=wmin:wamount:wmax;
    B=-w^2*Mass_Matrix+j*w*Damper_Matrix+Spring_Matrix;
    X0(:,:,k)=B\A;
    Acc(:,:,k)=-w^2*X0(:,:,k);
    freq(k)=w;
    k=k+1;
end;


[x,y,z]=size (Acc);


for u=1:1:z;
    X1 (u)=Acc(1,1,u);
    X2 (u)=Acc(2,1,u);
    X3 (u)=Acc(3,1,u);
end;


freq_hz=freq/(2*pi);


magnitudeX1=abs(X1);
```

```
magnitudeX2=abs(X2);

magnitudeX3=abs(X3);


figure (1);
subplot (3,1,1);
plot (freq_hz,magnitudeX1,'b');
xlabel ('Frequency (Hz)');
ylabel ('Magnitude of Acceleration (m/s^2)');
title ('Front Unsprung Mass');

subplot (3,1,2);
plot (freq_hz,magnitudeX2,'b');
xlabel ('Frequency (Hz)');
ylabel ('Magnitude of Acceleration (m/s^2)');
title ('Rear Unsprung Mass');

subplot (3,1,3);
plot (freq_hz,magnitudeX3,'b');
xlabel ('Frequency (Hz)');
ylabel ('Magnitude of Accerleration (m/s^2)');
title ('Sprung Mass');
```

Appendix H: Damping calculations for the Quarter Car Model

$$c = \zeta \times c_{crit}$$

$$c_{crit} = 2\sqrt{m(K_T + K_W)}$$

$$\zeta = 0.15$$

Damping of front unsprung mass:

Stiff spring

c_{crit} = 5093.9 Ns/m

c = 0.15 x 5093.9 = 764.1 Ns/m

Combination 2

c_{crit} = 3766.9 Ns/m

c = 0.15 x 3766.9 = 565.0 Ns/m

Damping of rear unsprung mass:

Stiff spring

c_{crit} = 5093.9 Ns/m

c = 0.15 x 5093.9 = 764.1 Ns/m

Combination 2

c_{crit} = 3846.7 Ns/m

c = 0.15 x 3846.7 = 577.0 Ns/m

Appendix I: MATLAB Code for Quarter Car Model Vertical Accelerations

Below is the MATLAB code for determining the vertical accelerations of the unsprung and sprung masses in the Quarter Car Model.

```
%Quarter Car Model - Acceleration
%Peter Corke 20139918

clear;

m=;           % unsprung mass [kg]
M=;         % sprung mass at one wheel[kg]
c=;           % wheel damping rate [Ns/m]
kt=;        % Tyre spring rate [N/m]
kw=;       % Wheel spring rate [N/m]
A1=;          % Amplitude [m]
hzmin=;       % Frequency range [Hz]
hzmax=;
n=1000;
hzamount=(hzmax-hzmin)/(n-1);

Mass_Matrix=[m,0;0,M];
Damper_Matrix=[c,-c;-c,c];
Spring_Matrix=[kt+kw,-kw;-kw,kw];
A=[A1;0];

wmin=hzmin*2*pi;
wmax=hzmax*2*pi;
wamount=hzamount*2*pi;
k=1;

for w=wmin:wamount:wmax;
    B=-w^2*Mass_Matrix+j*w*Damper_Matrix+Spring_Matrix;
    X0(:,:,k)=B\A;;
    Acc(:,:,k)=-w^2*X0 (:,:,k);
    freq (k)=w;
    k=k+1;
end;

[x,y,z]=size (Acc);

for u=1:1:z;
    X1 (u)=Acc (1,1,u);
    X3 (u)=Acc (2,1,u);
end;
```

```
freq_hz=freq/(2*pi);

magnitudeX1=1000*abs (X1);

magnitudeX3=1000*abs (X3);

figure (1);
subplot (2,1,1);
plot (freq_hz,magnitudeX1,'b');
xlabel ('Frequency (Hz)');
ylabel ('Magnitude of Acceleration (mm/s^2)');
title ('Unsprung Mass');

subplot (2,1,2);
plot (freq_hz,magnitudeX3,'b');
xlabel ('Frequency (Hz)');
ylabel ('Magnitude of Acceleration (mm/s^2)');
title ('Sprung Mass');
```